

① Define a function $f: \mathbb{R} \rightarrow \mathbb{R}$ by $f(x) = 3x + 7$.

(a) True or False:

(i) f is onto
(2 pts) True

(ii) $(4, -1) \in f^{-1}$
(2 pts) True

(b) Find an explicit formula for $f^{-1}(x)$.

(4 pts)

$$\begin{aligned} f^{-1} &= \{(y, x) : (x, y) \in f\} \\ &= \{(y, x) : y = 3x + 7\} \\ &= \{(x, y) : x = 3y + 7\} \\ &= \{(x, y) : y = \frac{x-7}{3}\}. \end{aligned}$$

$$f^{-1}(x) = \frac{x-7}{3}$$

(c) Prove that f is one-to-one.

(4 pts) Need to prove that $f(x) = f(y)$ implies $x = y$.

Assume $f(x) = f(y)$.

$$\therefore 3x + 7 = 3y + 7$$

$$\therefore 3x = 3y$$

$$\therefore x = y$$

$\therefore f$ is one-to-one.

Q.E.D.
