Solutions to Exam 1

(6)(a)(b) are 6 pts each. Every other part of every other problem is 11 pts. 100 pts total.

(1)

P	Q	$P \Rightarrow Q$	$\sim(P \Rightarrow Q)$	$P \wedge (\sim Q)$
T	T	T	F	F
T	F	F	T	T
F	T	T	F	F
F	F	T	F	F

same

(2) $\sim[(P \wedge (\sim Q)) \vee (\sim R)]$

$\equiv (\sim(P \wedge (\sim Q))) \wedge (\sim(\sim R))$

$\equiv (\sim P \vee (\sim(\sim Q))) \wedge R$

$\equiv (\sim P \vee Q) \wedge R$

(3) For every positive real number there exists a smaller positive real number.

(4)(a) $\exists x (x \text{ is a liar}) \wedge \sim \forall x (x \text{ is a liar})$

$\equiv \exists x (x \text{ is a liar}) \wedge \exists x (x \text{ is not a liar})$

(4)(b) Either no one is a liar, or everyone is a liar.

(5)(a) Suppose a divides b. Then $b = ak$ for some integer k.

$\therefore\ bc = (ak)c = a(kc)$.

kc is an integer, so a divides bc. Q.E.D.

Solution to BONUS!

Suppose $\log_{10} 2$ is rational. Then there are positive integers a and b such that

$\log_{10} 2 = \frac{a}{b}$

$\therefore\ 2 = 10^{\frac{a}{b}}$

$\therefore\ 2^b = 10^a$

(5)(b) Suppose a and b are odd. Then $a = 2k+1$ for some integer k, and $b = 2l+1$ for some integer l.

$\therefore\ a+b = (2k+1)+(2l+1)$

$= 2k+2l+2$

$= 2(k+l+2)$

$\therefore\ a+b$ is even. Q.E.D.

(6)(a) The contrapositive of 5(a) is true because it is equivalent to 5(a), which is true.

(b) The converse of 5(b) is false, because a & b could both be even and then the sum is even too. E.g., $6+4=10$ which is even, while neither 6 nor 4 is odd.

(7) Suppose $\frac{n}{n+1} \geq \frac{n+1}{n+2}$.

Then multiplying both sides by the positive number ~~[illegible]~~ $(n+1)(n+2)$, we get

$n(n+2) \geq (n+1)^2$

$\therefore\ n^2+2n \geq n^2+2n+1$

$\therefore\ (n^2+2n) - (n^2+2n) \geq n^2+2n+1 - (n^2+2n)$

$\therefore\ 0 \geq 1$

This is a contradiction, so

$\frac{n}{n+1} < \frac{n+1}{n+2}$ Q.E.D.

$\therefore\ 2^b = (2 \cdot 5)^a = 2^a \cdot 5^a$.

$\therefore\ 5$ divides 2^b, contradicting uniqueness of prime decomposition.

$\therefore\ \log_{10} 2$ is irrational. Q.E.D.