MH 2660   Quiz #9   12/12   Name: The Prof

[Also please write your name on the back of this sheet]

(1) Let $\vec{u}_1 = \begin{bmatrix} 3/5 \\ 0 \\ 4/5 \end{bmatrix}$, $\vec{u}_2 = \begin{bmatrix} 4/5 \\ 0 \\ -3/5 \end{bmatrix}$, $\vec{u}_3 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$.

(a) Show that $\vec{u}_1, \vec{u}_2, \vec{u}_3$ is an orthonormal basis for $\mathbb{R}^3$.

(4pts) $\|\vec{u}_1\| = \sqrt{9/25 + \frac{16}{25}} = \sqrt{\frac{25}{25}} = 1$, $\|\vec{u}_2\| = \sqrt{\frac{16}{25} + \frac{9}{25}} = 1$,

$\|\vec{u}_3\| = \sqrt{0^2+1^2+0^2} = 1.$

$\vec{u}_1 \cdot \vec{u}_2 = \frac{12}{25} - \frac{12}{25} = 0$, $\vec{u}_1 \cdot \vec{u}_3 = 0$, $\vec{u}_2 \cdot \vec{u}_3 = 0$.

Their lengths are 1, and any two are $\perp$, so they are orthonormal.

(b) Write $\vec{v} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ as a linear combination of $\vec{u}_1, \vec{u}_2, \& \vec{u}_3$.

(4pts) $\vec{v} = \langle \vec{v}, \vec{u}_1 \rangle \vec{u}_1 + \langle \vec{v}, \vec{u}_2 \rangle \vec{u}_2 + \langle \vec{v}, \vec{u}_3 \rangle \vec{u}_3$

$= (3/5 + \frac{12}{5}) u_1 + (\frac{4}{5} - \frac{9}{5}) \vec{u}_2 + 2\vec{u}_3$

$= \underline{3\vec{u}_1 - \vec{u}_2 + 2\vec{u}_3}$

(2) $\vec{v}_1 = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$ and $\vec{v}_2 = \begin{bmatrix} 5 \\ 10 \end{bmatrix}$ are a basis for $\mathbb{R}^2$. Use the Gram-Schmidt process to convert $\vec{v}_1, \vec{v}_2$ to an orthonormal basis.

(4pts) $\vec{u}_1 = \frac{\vec{v}_1}{\|\vec{v}_1\|} = \frac{\vec{v}_1}{\sqrt{16+9}} = \frac{\vec{v}_1}{5} = \begin{bmatrix} 4/5 \\ 3/5 \end{bmatrix}$.

$\vec{p}_1 = \langle \vec{v}_2, \vec{u}_1 \rangle \vec{u}_1 = (4+6)\vec{u}_1 = 10\vec{u}_1 = \begin{bmatrix} 8 \\ 6 \end{bmatrix}$.

$\vec{v}_2 - \vec{p}_1 = \begin{bmatrix} -3 \\ 4 \end{bmatrix}$ $\|v_2 - p_1\| = \sqrt{9+16} = 5$

$\vec{u}_2 = \frac{\vec{v}_2 - \vec{p}_1}{\|\vec{v}_2 - p_1\|} = \frac{\begin{bmatrix} -3 \\ 4 \end{bmatrix}}{5} = \begin{bmatrix} -3/5 \\ 4/5 \end{bmatrix}$

$\vec{u}_1 = \begin{bmatrix} 4/5 \\ 3/5 \end{bmatrix}$, $\vec{u}_2 = \begin{bmatrix} -3/5 \\ 4/5 \end{bmatrix}$