MH 2660 Quiz #8 $\frac{12}{12}$ Name: The Prof

Also please write your name on the back of this sheet near the top.

① Given data points $(x_1, y_1) = (0,0)$, $(x_2, y_2) = (1,1)$, $(x_3, y_3) = (2,3)$, and $(x_4, y_4) = (3,4)$, the problem of finding the best fitting line of the form $c_0 + c_1 x = y$ is equivalent to finding the least squares sol'n of $A\vec{c} = \vec{y}$, where $\vec{c} = \begin{bmatrix} c_0 \\ c_1 \end{bmatrix}$, and

$$A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix} \quad \text{and} \quad \vec{y} = \begin{bmatrix} 0 \\ 1 \\ 3 \\ 4 \end{bmatrix}.$$

(3pts) (2pts)

Fill in the blanks. You need not solve the system.

② Define an inner product on $\mathbb{R}^3$ by $\langle \vec{x}, \vec{y} \rangle = 2x_1y_1 + 3x_2y_2 + x_3y_3$. Let $\vec{v} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ and $\vec{w} = \begin{bmatrix} -1 \\ 0 \\ 4 \end{bmatrix}$. Find, with respect to this inner product,

(a) $\langle \vec{v}, \vec{w} \rangle$

(2pts) $\langle \vec{v}, \vec{w} \rangle = 2 \cdot 1 \cdot (-1) + 3 \cdot 2 \cdot 0 + 3 \cdot 4 = -2 + 12 = 10$

(b) $\|\vec{v}\|$

(2pts) $\|\vec{v}\| = \sqrt{\langle \vec{v}, \vec{v} \rangle} = \sqrt{2 \cdot 1^2 + 3 \cdot 2^2 + 3^2} = \sqrt{23}$

(c) the vector projection of $\vec{w}$ on $\vec{v}$

(3pts) $\vec{p} = \dfrac{\langle \vec{w}, \vec{v} \rangle}{\langle \vec{v}, \vec{v} \rangle} \vec{v} = \dfrac{10}{23} \vec{v} = \begin{bmatrix} 10/23 \\ 20/23 \\ 30/23 \end{bmatrix}$