

① Find the standard matrix representation of

Please also write your name on the back of this sheet near the top.

(a) the transformation L from \mathbb{R}^2 to \mathbb{R}^3 defined by $L(\vec{x}) = \begin{bmatrix} x_1^2 \\ x_1 - x_2 \\ 3x_1 \end{bmatrix}$

(2 pts) $L([b]) = \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix}$ $L([c]) = \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}$

$$A = \begin{bmatrix} 0 & 2 \\ 1 & -1 \\ 3 & 0 \end{bmatrix}$$

(b) the transformation from \mathbb{R}^2 to \mathbb{R}^2 which first rotates a vector 90° clockwise, and then reflects it about the x_2 -axis.

(2 pts) Method I: $L([e_1]) = [e_2] \xrightarrow{\text{rotate}} [e_1] \xrightarrow{\text{reflect}} [e_1]$

$L([e_2]) = [e_1] \xrightarrow{\text{rotate}} [e_2] \xrightarrow{\text{reflect}} [-e_2]$

$$A = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$$

Method II: Rotation matrix $B: L([e_1]) = [e_2], L([e_2]) = [-e_1]$

$$B = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

Answer: $A = CB$

$$A = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$$

② Let $\vec{x} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $\vec{y} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$.

(a) Find $\|\vec{x}\|$ and $\|\vec{y}\|$.

(2 pts) $\|\vec{x}\| = \sqrt{1+4} = \sqrt{5} = 3$

$\|\vec{y}\| = \sqrt{9+16} = \sqrt{25} = 5$

(b) Find the cosine of the angle between \vec{x} and \vec{y} .

(2 pts) $\vec{x} \cdot \vec{y} = \|\vec{x}\| \|\vec{y}\| \cos \theta$

$$3 \cdot 1 + 2 \cdot 2 + (-2) \cdot 4 = 3 \cdot 5 \cos \theta$$

$$3 + 0 - 8 = 15 \cos \theta$$

$$-5 = 15 \cos \theta$$

$$\cos \theta = -\frac{5}{15} = -\frac{1}{3}$$

(c) Find the scalar projection of \vec{x} on \vec{y} .

(2 pts) $\frac{\vec{x} \cdot \vec{y}}{\|\vec{y}\|} = \frac{-5}{5} = -1$

(d) Find the vector projection of \vec{x} on \vec{y} .

(2 pts) $\frac{\vec{x} \cdot \vec{y}}{\vec{y} \cdot \vec{y}} \vec{y} = \frac{-5}{25} \begin{bmatrix} 3 \\ 0 \\ 4 \end{bmatrix} = -\frac{1}{5} \begin{bmatrix} 3 \\ 0 \\ 4 \end{bmatrix}$

$$\begin{bmatrix} -3/5 \\ 0 \\ -4/5 \end{bmatrix}$$