MH 2660 Quiz #6 1/12

Name: The Prof

Also please write your name on the back of this sheet near the top.

① Show that the transformation $L: \mathbb{R}^2 \to \mathbb{R}^3$ defined by $L\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} 3x_1 \\ 0 \\ x_1+x_2 \end{bmatrix}$ is linear.

(5pts) $L(\alpha\vec{x}+\beta\vec{y}) = L\left(\begin{bmatrix} \alpha x_1+\beta y_1 \\ \alpha x_2+\beta y_2 \end{bmatrix}\right)$

$$= \begin{bmatrix} 3(\alpha x_1+\beta y_1) \\ 0 \\ (\alpha x_1+\beta y_1)+(\alpha x_2+\beta y_2) \end{bmatrix}$$

$$= \begin{bmatrix} 3\alpha x_1+3\beta y_1 \\ 0 \\ (\alpha x_1+\alpha x_2)+(\beta y_1+\beta y_2) \end{bmatrix} = \begin{bmatrix} 3\alpha x_1 \\ 0 \\ \alpha x_1+\alpha x_2 \end{bmatrix} + \begin{bmatrix} 3\beta y_1 \\ 0 \\ \beta y_1+\beta y_2 \end{bmatrix}$$

$$= \alpha\begin{bmatrix} 3x_1 \\ 0 \\ x_1+x_2 \end{bmatrix} + \beta\begin{bmatrix} 3y_1 \\ 0 \\ y_1+y_2 \end{bmatrix} = \alpha L(\vec{x}) + \beta L(\vec{y})$$

② Let $A = \begin{bmatrix} 1 & 2 & 0 & 3 \\ 2 & 4 & 1 & 4 \end{bmatrix}$. Find bases for (a) the null space; (b) the row space; and (c) the column space.

(a) (3pts) $\begin{bmatrix} 1 & 2 & 0 & 3 \\ 2 & 4 & 1 & 4 \end{bmatrix} \xrightarrow{-2r_1+r_2} \begin{bmatrix} 1 & 2 & 0 & 3 \\ 0 & 0 & 1 & -2 \end{bmatrix}$

x_2, x_4 free. Let $x_2=\alpha$, $x_4=\beta$.

$x_1+2x_2+3x_4=0 \Rightarrow x_1=-2x_2-3x_4=-2\alpha-3\beta$

$x_3-2x_4=0 \Rightarrow x_3=2x_4=2\beta$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -2\alpha-3\beta \\ \alpha \\ 2\beta \\ \beta \end{bmatrix} = \begin{bmatrix} -2\alpha \\ \alpha \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} -3\beta \\ 0 \\ 2\beta \\ \beta \end{bmatrix} = \alpha\begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \end{bmatrix} + \beta\begin{bmatrix} -3 \\ 0 \\ 2 \\ 1 \end{bmatrix}$$

Basis for null space: $\begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 0 \\ 2 \\ 1 \end{bmatrix}$

(b) (2pts) Basis for row space: $[1\ 2\ 0\ 3], [0\ 0\ 1\ -2]$

(c) (2pts) Basis for column space: $\begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

because the leading ones of row echelon form are in columns 1 and 3.