MH 266C Quiz #5 $\frac{12}{12}$ Name: The Prof

Please also write your name on the back of this sheet near the top.

① Let $\vec{x} = \begin{bmatrix} -1 \\ 4 \end{bmatrix}$. Find the coordinates of $\vec{x}$ with respect to the basis $\vec{u}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \vec{u}_2 = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ for $\mathbb{R}^2$.

(3pts) The answer is $\mathcal{U}^{-1}\vec{x}$, where $\mathcal{U} = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$. So $\mathcal{U}^{-1} = \frac{1}{-1}\begin{bmatrix} 3 & -2 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} -3 & 2 \\ 2 & -1 \end{bmatrix}$

$$\mathcal{U}^{-1}\vec{x} = \begin{bmatrix} -3 & 2 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} -1 \\ 4 \end{bmatrix} = \begin{bmatrix} 11 \\ -6 \end{bmatrix}$$ ← coordinates of $\vec{x}$ w.r.t. $\vec{u}_1, \vec{u}_2$.

(This means $\vec{x} = 11\vec{u}_1 - 6\vec{u}_2$.)

② Let $\vec{u}_1, \vec{u}_2$ be as in Problem ①, and let $\vec{v}_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$.

(a) Find the transition matrix from $\vec{v}_1, \vec{v}_2$ to $\vec{u}_1, \vec{u}_2$.

(3pts) The answer is $\mathcal{U}^{-1}V$, where $\mathcal{U}$ is as in ① and $V = \begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix}$

$$\mathcal{U}^{-1}V = \begin{bmatrix} -3 & 2 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix} = \begin{bmatrix} -5 & 2 \\ 3 & 0 \end{bmatrix}$$

(b) Use your answer to (a) to find the coordinates of $\vec{y} = \vec{v}_1 + 4\vec{v}_2$ with respect to $\vec{u}_1, \vec{u}_2$.

(3pts) The answer is $\mathcal{U}^{-1}V\begin{bmatrix} 1 \\ 4 \end{bmatrix} = \begin{bmatrix} -5 & 2 \\ 3 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 4 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$

(so $\vec{y} = 3\vec{u}_1 + 3\vec{u}_2$)

③ Let $A = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 2 \\ 3 & -2 & 4 \end{bmatrix}$ be the transition matrix from the basis $\vec{x}_1, \vec{x}_2, \vec{x}_3$ for $\mathbb{R}^3$ to the basis $\vec{y}_1, \vec{y}_2, \vec{y}_3$. If $\vec{w} = 2\vec{x}_1 - 3\vec{x}_2 + \vec{x}_3$, write $\vec{w}$ as a linear combination of $\vec{y}_1, \vec{y}_2, \vec{y}_3$.

(3pts) ~~The answer~~ $A\begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 2 \\ 3 & -2 & 4 \end{bmatrix}\begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix} = \begin{bmatrix} -5 \\ -1 \\ 16 \end{bmatrix}$

Thus $\begin{bmatrix} -5 \\ -1 \\ 16 \end{bmatrix}$ are the coordinates of $\vec{w}$ w.r.t. $\vec{y}_1, \vec{y}_2, \vec{y}_3$,

and so $\vec{w} = -5\vec{y}_1 - \vec{y}_2 + 16\vec{y}_3$

In problems ① & ②, you may use the formula

$$A^{-1} = \frac{1}{\det A}\begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix}$$ (where $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$)