MH 2660 Quiz #4 12/12 Name: The Prof

(Please also write your name on the back of this sheet near the top.)

① Determine whether or not the following subsets of $\mathbb{R}^2$ are subspaces of $\mathbb{R}^2$ (give reasoning/show work).

(a) $S = \left\{ \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} : x_1 - x_2 = 1 \right\}$

(3pts) Possible answer I: No, because $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ is not in S.

Possible answer II: No. $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ is in S since $x_1 - x_2 = 1 - 0 = 1$,
but $2\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$ is not in S, since $2 - 0 = 2 \neq 1$.
So S is not closed under scalar mult.

Possible answer III:
No. $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ is in S but $\begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$ is not in S.
So S is not closed under addition.

(b) $S = \left\{ \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} : x_2 = 3x_1 \right\}$ = all vectors of form $\begin{bmatrix} x \\ 3x \end{bmatrix}$.

(3 pts) Check (i). $\alpha \begin{bmatrix} x \\ 3x \end{bmatrix} = \begin{bmatrix} \alpha x \\ 3\alpha x \end{bmatrix}$. This is in S since 2nd coörd = 3 times 1st coörd.

So S is closed under scalar mult.

Check (ii). $\begin{bmatrix} x \\ 3x \end{bmatrix} + \begin{bmatrix} y \\ 3y \end{bmatrix} = \begin{bmatrix} x+y \\ 3x+3y \end{bmatrix} = \begin{bmatrix} x+y \\ 3(x+y) \end{bmatrix}$, which is in S.

So S is closed under addition.

S is closed under scalar mult. and under addition,
so S is a subspace of $\mathbb{R}^2$

② Let $\vec{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$, $\vec{v}_2 = \begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix}$, $\vec{v}_3 = \begin{bmatrix} 3 \\ -2 \\ 2 \end{bmatrix}$.

(a) Do $\vec{v}_1, \vec{v}_2$ span $\mathbb{R}^3$? (Give reasoning/show work)

(3pts) No, 2 vectors can't span the 3 dimensional space $\mathbb{R}^3$
(they only span some plane in $\mathbb{R}^3$)

(b) Do $\vec{v}_1, \vec{v}_2, \vec{v}_3$ span $\mathbb{R}^3$? (Give reasoning/show work)

(3pts) $\begin{vmatrix} 1 & -1 & 3 \\ 0 & 2 & -2 \\ 1 & 0 & 2 \end{vmatrix} = \begin{vmatrix} 2 & -2 \\ 0 & 2 \end{vmatrix} + \begin{vmatrix} -1 & 3 \\ 2 & -2 \end{vmatrix} = 4 + (2-6)$
$= 0$

No, they do not span $\mathbb{R}^3$