MH 2660 Exam 1

1. Consider the following augmented matrices:

$$\text{(i)}\begin{bmatrix} 1 & 3 & 0 & : & 4 \\ 0 & 0 & 1 & : & 0 \\ 0 & 0 & 0 & : & 0 \end{bmatrix};\ \text{(ii)}\begin{bmatrix} 1 & 3 & 0 & : & 4 \\ 0 & 2 & 0 & : & 3 \\ 0 & 0 & 0 & : & 1 \end{bmatrix};\ \text{(iii)}\begin{bmatrix} 1 & 0 & 3 & : & 4 \\ 0 & 1 & 0 & : & 2 \\ 0 & 0 & 1 & : & 2 \end{bmatrix}.$$

(a) Which of (i),(ii), and (iii) are in row echelon form?
(b) Which are in reduced row echelon form?
(c) Which correspond to an inconsistent system of equations?

2. Find $3A^TB$ if $A = \begin{bmatrix} 2 & 1 \\ -1 & 0 \\ 3 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ 5 & -3 \\ 0 & 4 \end{bmatrix}$.

3. For the following system, write the augmented matrix, get it in row echelon form or reduced row echelon form, and then solve the system:

$$\begin{aligned} 2x_1 + 8x_2 - x_3 + 3x_4 &= 3 \\ x_1 + 4x_2 + x_3 + 4x_4 &= -1 \\ x_1 + 4x_2 - 2x_3 \qquad &= 3 \end{aligned}$$

4. The system of problem 3 is equivalent to the matrix equation $A\vec{x} = \vec{b}$, where $A =???$, $\vec{x} =???$, and $\vec{b} =???$.

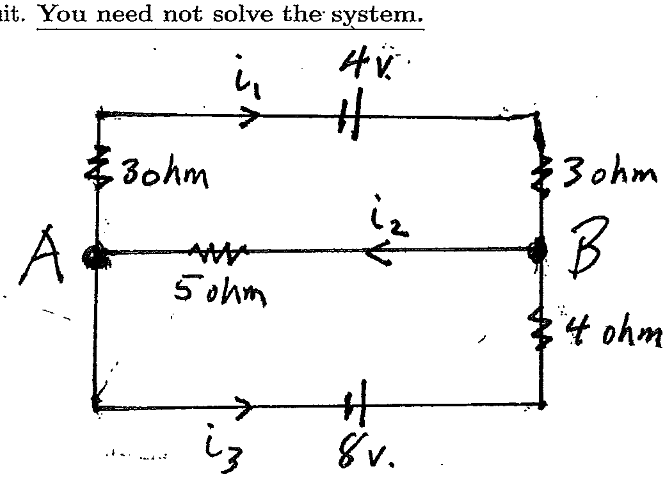
5. Find A^{-1} if $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 2 & 2 & -2 \end{bmatrix}$.

6. Let A and B be $n \times n$ matrices. True or False (call it false if it is not **always** true):
(a) $(A+B)^2 = A^2 + 2AB + B^2$;
(b) If $AB = AC$, then $B = C$.
(c) If det $A \neq 0$, then A^{-1} must exist.

7. Find det(A) if $A = \begin{bmatrix} 2 & 1 & 1 & 0 \\ 0 & 0 & -1 & 2 \\ 1 & 0 & 3 & -2 \\ 4 & 2 & 3 & 5 \end{bmatrix}$

8. Solve algebraically for X: $4X = XA + 2B$.

9. Write the system of equations you would solve to find the currents in the following circuit. **You need not solve the system.**



MA 2660 Sol'ns to Exam 1

① (a)(4pts) (i) a (iii)
(b)(4pts) (i) (c)(4pts) (ii)

② (10pts)
$$3A^TB = 3\begin{bmatrix}2&-1&3\\1&0&-1\end{bmatrix}\begin{bmatrix}1&2\\5&-3\\0&4\end{bmatrix}$$
$$=\begin{bmatrix}6&-3&9\\3&0&-3\end{bmatrix}\begin{bmatrix}1&2\\5&-3\\0&4\end{bmatrix}=\begin{bmatrix}-9&57\\3&-6\end{bmatrix}$$

③ (15pts)
$$\left[\begin{array}{cccc|c}2&8&-1&3&3\\1&4&1&4&-1\\1&4&-2&0&3\end{array}\right]\to\left[\begin{array}{cccc|c}1&4&1&4&-1\\2&8&-1&3&3\\1&4&-2&0&3\end{array}\right]$$
$$\xrightarrow{-2r_1+r_2 \quad -r_1+r_3}\left[\begin{array}{cccc|c}1&4&1&4&-1\\0&0&-3&-5&5\\0&0&-3&-4&4\end{array}\right]\xrightarrow[-r_3]{-r_2}\left[\begin{array}{cccc|c}1&4&1&4&-1\\0&0&3&5&-5\\0&0&3&4&-4\end{array}\right]$$
$$\xrightarrow{-r_2+r_3}\left[\begin{array}{cccc|c}1&4&1&4&-1\\0&0&3&5&-5\\0&0&0&-1&1\end{array}\right]\xrightarrow[5r_3+r_2]{4r_3+r_1}\left[\begin{array}{cccc|c}1&4&1&0&3\\0&0&3&0&0\\0&0&0&-1&1\end{array}\right]$$
$$\xrightarrow{-r_3 \text{ and } \frac{1}{3}r_2}\left[\begin{array}{cccc|c}1&4&1&0&3\\0&0&1&0&0\\0&0&0&1&-1\end{array}\right]\xrightarrow{-r_2+r_1}\left[\begin{array}{cccc|c}1&4&0&0&3\\0&0&1&0&0\\0&0&0&1&-1\end{array}\right]$$

x_2 free, let $x_2=\alpha$

$x_4=-1$ $x_3=0$ $x_1=3-4x_2=3-4\alpha$

④ (4pts)
$$A=\begin{bmatrix}2&8&-1&3\\1&4&1&4\\1&4&-2&0\end{bmatrix},\ \vec{b}=\begin{bmatrix}3\\-1\\3\end{bmatrix},\ \vec{x}=\begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix}$$

⑤ (10pts)
$$\left[\begin{array}{ccc|ccc}0&1&1&1&0&0\\1&1&0&0&1&0\\2&2&-2&0&0&1\end{array}\right]\to\left[\begin{array}{ccc|ccc}1&1&0&0&1&0\\0&1&1&1&0&0\\2&2&-2&0&0&1\end{array}\right]$$
$$\xrightarrow{-2r_1+r_3}\left[\begin{array}{ccc|ccc}1&1&0&0&1&0\\0&1&1&1&0&0\\0&0&-2&0&-2&1\end{array}\right]\xrightarrow{-\frac{1}{2}r_3}\left[\begin{array}{ccc|ccc}1&1&0&0&1&0\\0&1&1&1&0&0\\0&0&1&0&1&-\frac{1}{2}\end{array}\right]$$
$$\xrightarrow{-r_3+r_2}\left[\begin{array}{ccc|ccc}1&1&0&0&1&0\\0&1&0&1&-1&\frac{1}{2}\\0&0&1&0&1&-\frac{1}{2}\end{array}\right]\xrightarrow{-r_2+r_1}\left[\begin{array}{ccc|ccc}1&0&0&-1&2&-\frac{1}{2}\\0&1&0&1&-1&\frac{1}{2}\\0&0&1&0&1&-\frac{1}{2}\end{array}\right]$$
$$\begin{bmatrix}-1&2&-\frac{1}{2}\\1&-1&\frac{1}{2}\\0&1&-\frac{1}{2}\end{bmatrix}=A^{-1}$$

⑥ (4pts each) (a) F (b) F (c) T

⑦ (12pts) expand along row 2
$$\begin{vmatrix}2&1&1&0\\0&0&-1&2\\1&0&3&-2\\4&2&3&5\end{vmatrix}=-0+0-(-1)\underbrace{\begin{vmatrix}2&1&0\\1&0&-2\\4&2&5\end{vmatrix}}_{\text{row 1}}+2\cdot\underbrace{\begin{vmatrix}2&1&1\\1&0&3\\4&2&3\end{vmatrix}}_{\text{row 2}}$$
$$=\left(2\begin{vmatrix}0&-2\\2&5\end{vmatrix}-\begin{vmatrix}1&-2\\4&5\end{vmatrix}\right)+2\left[-1\cdot\begin{vmatrix}1&1\\2&3\end{vmatrix}+0-3\cdot\begin{vmatrix}2&1\\4&2\end{vmatrix}\right]$$
$$=\left(2(0-(-4))-(5-(-8))\right)+2\left[-(3-2)-3(4-4)\right]$$
$$=(8-13)+2\left[-1+0\right]=-5-2=-7$$

⑧ (10pts)
$$4X=XA+2B$$
$$4X-XA=2B$$
$$X(4I-A)=2B$$
$$X\underbrace{(4I-A)(4I-A)^{-1}}_{I}=2B(4I-A)^{-1}$$
$$XI= \text{''}$$
$$X=2B(4I-A)^{-1}$$

⑨ (10pts) Node A: $i_2=i_1+i_3$
Node B: $i_2=i_1+i_3$

Top loop: $3i_1+3i_1+5i_2=4$
$6i_1+5i_2=4$

Bottom: $5i_2+4i_3=8$

The system:
$$\boxed{\begin{aligned}i_1-i_2+i_3&=0\\6i_1+5i_2&=4\\5i_2+4i_3&=8\end{aligned}}$$