MATH-1150 PRACTICE Exam #5 Instructor: Eric Harshbarger

Student Name: KEY

Show all relevant work (use back of pages for scratch paper, if needed). **CIRCLE FINAL ANSWERS.**

1. [5 pts each] Evaluate the following and express answers in standard form *a* + *bi*:

a) $(2 - 4i) - (5 + 8i) = \boxed{-3 - 12i}$

b) $\dfrac{7-3i}{4+2i} \cdot \dfrac{4-2i}{4-2i} = \dfrac{28-14i-12i+6i^2}{16-8i+8i-4i^2} = \dfrac{28-6-26i}{16+4} = \dfrac{22-26i}{20} = \dfrac{22}{20} - \dfrac{26}{20}i$

$\boxed{\dfrac{11}{10} - \dfrac{13}{10}i}$

c) $(2 + 7i)(1 - 3i) = 2 - 6i + 7i - 21i^2 = 2 + 21 + i = \boxed{23 + i}$

d) $i^{63} = i^{60} \cdot i^3 = (i^4)^{15} \cdot i^2 \cdot i^1 = 1^{15}(-1)i = \boxed{-i}$

2. [5 pts] Find all solutions of the equation $\underset{a}{3x^2} + \underset{b}{2x} + \underset{c}{7} = 0$ and express them in the form *a* + *bi*.

$x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x = \dfrac{-2 \pm \sqrt{2^2-4(3)(7)}}{2(3)} = \dfrac{-2 \pm \sqrt{4-84}}{6}$

$= \dfrac{-2 \pm \sqrt{-80}}{6} = \dfrac{-2 \pm 4i\sqrt{5}}{6} = \dfrac{-2}{6} \pm \dfrac{4i\sqrt{5}}{6} = \boxed{\dfrac{-1}{3} \pm \dfrac{2i\sqrt{5}}{3}}$

3. [5 pts] Convert the polar coordinates $(8, -\frac{\pi}{6})$ to rectangular coordinates (exact answer, no decimals).

$r \quad \theta$

$-30°$

$330°$

$x = r\cos\theta = 8\cos 330° = 8\left(\frac{\sqrt{3}}{2}\right) = 4\sqrt{3}$

$y = r\sin\theta = 8\sin 330° = 8\left(-\frac{1}{2}\right) = -4$

$\boxed{(4\sqrt{3}, -4)}$

$x \quad y$

4. [5 pts] Convert the rectangular equation $x^2 + y^2 = 16$ to polar form.

$x^2 + y^2 = 16$

$(r\cos\theta)^2 + (r\sin\theta)^2 = 16$

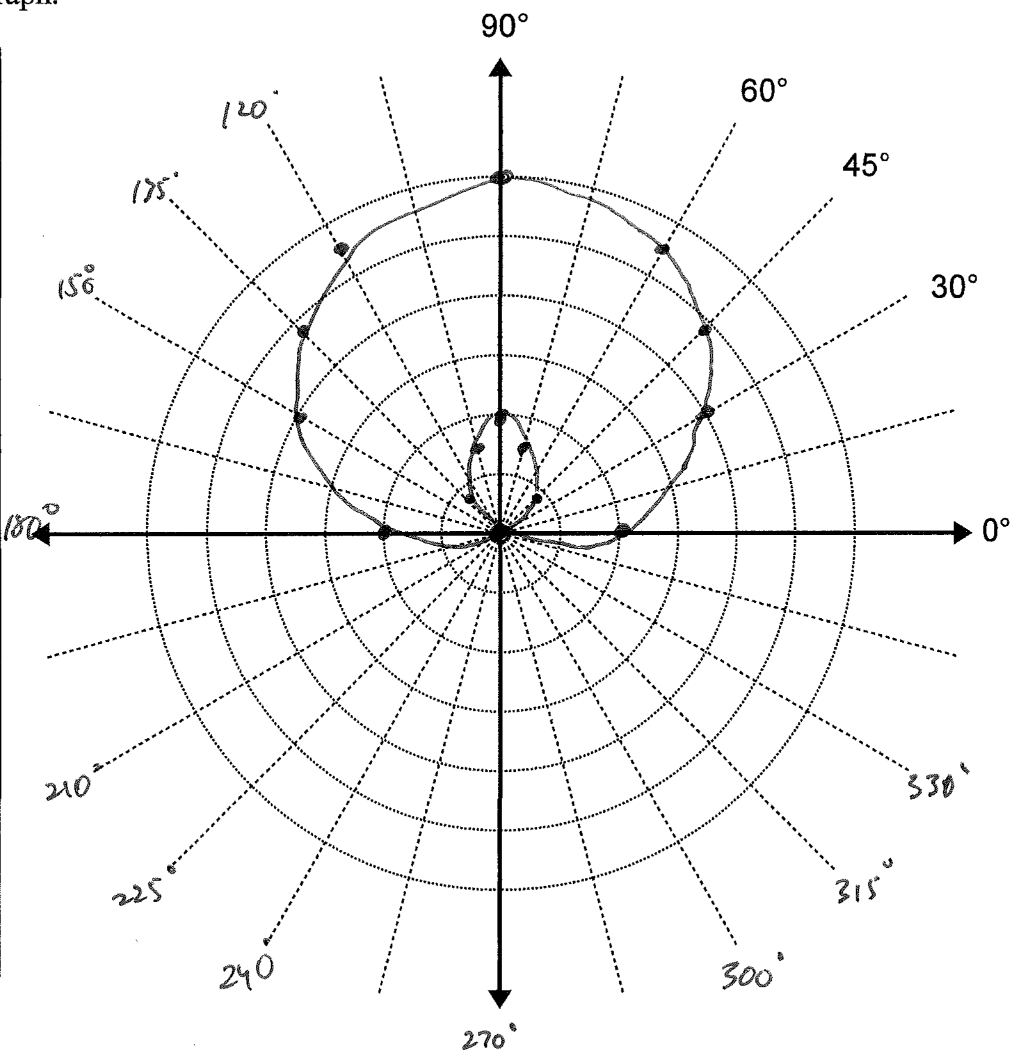
$r^2\cos^2\theta + r^2\sin^2\theta = 16$

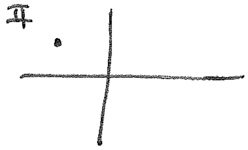
$r^2(\cos^2 + \sin^2\theta) = 16$

$r^2(1) = 16 \rightarrow \boxed{r^2 = 16}$

5. [20 pts.] Complete the table below by evaluating the equation: $r = 2 + 4\sin\theta$ (you may use a calculator and leave answers as decimals). Then, plot each of the points, and connect them in order to draw a sketch of the graph.

θ	r
0°	2
30°	4
45°	4.8
60°	5.5
90°	6
120°	5.5
135°	4.8
150°	4
180°	2
210°	0
225°	-0.8
240°	-1.5
270°	-2
300°	-1.5
315°	-0.8
330°	0
360°	2





6. [15 pts.] Calculate $\left(-\frac{\sqrt{3}}{2}+\frac{1}{2}i\right)^7$ (express answer in $a+bi$ form exactly — no decimals).

$r = \sqrt{\left(-\frac{\sqrt{3}}{2}\right)^2+\left(\frac{1}{2}\right)^2} = \sqrt{\frac{3}{4}+\frac{1}{4}} = \sqrt{\frac{4}{4}} = 1$

$\tan\theta = \frac{\frac{1}{2}}{-\frac{\sqrt{3}}{2}} = \frac{-1}{\sqrt{3}}$

$\bar{\theta} = 30^\circ$

$\theta = 150^\circ$

$\left(1\left(\cos 150^\circ + i\sin 150^\circ\right)\right)^7$

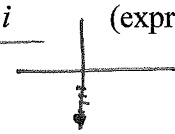
$1^7\left(\cos 7\cdot 150^\circ + i\sin 7\cdot 150^\circ\right)$

$1\left(\cos 1050^\circ + i\sin 1050^\circ\right)$

$\left(\cos 330^\circ + i\sin 330^\circ\right) = \boxed{\frac{\sqrt{3}}{2} - \frac{1}{2}i}$

7. [15 pts.] List below the five fifth roots of $0-32i$ (express answers in standard or polar form).

$n=5$



$z = 0-32i$

$r = \sqrt{0^2+(-32)^2} = \sqrt{32^2} = 32$

$\tan\theta = \frac{-32}{0}$

$\theta = 270^\circ$

$z = 32\left(\cos 270^\circ - i\sin 270^\circ\right)$

$32^{1/n} = 32^{1/5} = \sqrt[5]{32} = 2$

$\frac{360^\circ}{n} = \frac{360^\circ}{5} = 72^\circ$

$\frac{270^\circ}{n} = \frac{270^\circ}{5} = 54^\circ$

$w_0 = 2\left(\cos 54^\circ + i\sin 54^\circ\right) = 1.18 + 1.62i$

$w_1 = 2\left(\cos 126^\circ + i\sin 126^\circ\right) = -1.18 + 1.62i$

$w_2 = 2\left(\cos 198^\circ + i\sin 198^\circ\right) = -1.90 - 0.62i$

$w_3 = 2\left(\cos 270^\circ + i\sin 270^\circ\right) = -2i$

$w_4 = 2\left(\cos 342^\circ + i\sin 342^\circ\right) = 1.90 - 0.62i$

8. [5 pts. each part] Given vectors **u** = $\langle 3,-2 \rangle$, and **v** = $\langle 1,5 \rangle$:

a) find $5\mathbf{u} - 3\mathbf{v} =$ $5\langle 3,-2\rangle - 3\langle 1,5\rangle = \langle 15,-10\rangle + \langle -3,-15\rangle$

$\boxed{= \langle 12, -25 \rangle}$

b) find the magnitude of **u**: $|\mathbf{u}| =$

$|u| = \sqrt{3^2 + (-2)^2} = \sqrt{9+4} = \sqrt{13}$

c) find $\mathbf{u} \bullet \mathbf{v} =$ $3 \cdot 1 + (-2)(5) = 3 - 10 = \boxed{-7}$

d) calculate the angle between **u** and **v** (to the nearest degree)

$\cos\theta = \dfrac{u \cdot v}{|u||v|}$

$u \cdot v = -7$

$|u| = \sqrt{13}$

$|v| = \sqrt{1^2+5^2} = \sqrt{1+25} = \sqrt{26}$

$\cos\theta = \dfrac{-7}{\sqrt{13}\sqrt{26}}$

$\cos^{-1}(\cos\theta) = \cos^{-1}\left(\dfrac{-7}{\sqrt{13}\sqrt{26}}\right)$

$\boxed{\theta = 112.38^\circ}$