MATH-1150 PRACTICE Exam #5 Instructor: Eric Harshbarger

Student Name: KEY

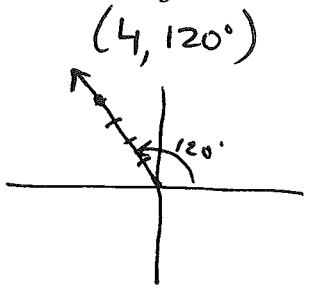
Show all relevant work (use back of pages for scratch paper, if needed). **CIRCLE FINAL ANSWERS.** All problems are worth 9 points except #6 which is worth 15 points.

1. Find the rectangular coordinates (exact, no decimals) for the point with polar coordinates $(4, \frac{2\pi}{3})$.

$x = 4\cos 120° = 4(-\frac{1}{2}) = -2$

$y = 4\sin 120° = 4(\frac{\sqrt{3}}{2}) = 2\sqrt{3}$

$\boxed{(-2, 2\sqrt{3})}$



2. Convert the polar equation $r = 3\csc\theta$ to a rectangular equation.

$r = 3\csc\theta$

$r = \frac{3}{\sin\theta}$

$r\sin\theta = 3$

$\boxed{y = 3}$

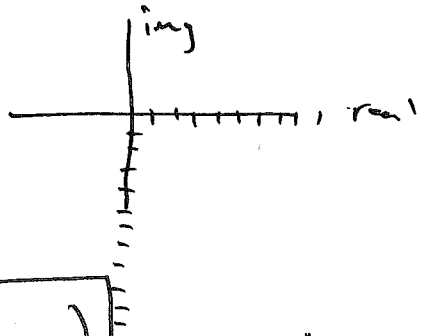
3. Write the complex number $9 - 12i$ in polar form.

$r = \sqrt{9^2 + (-12)^2} = \sqrt{81 + 144} = \sqrt{225} = 15$

$\tan\theta = \frac{-12}{9} = -\frac{4}{3}$

$\theta = \tan^{-1}(-\frac{4}{3})$

$\theta = -53.1°$ or $306.9°$



~~(15, 306.9°)~~

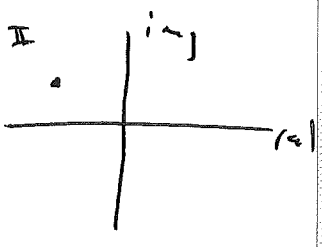
$\boxed{15(\cos 306.9° + i\sin 306.9°)}$

4. Given $z_1 = 12(\cos 125° + i \sin 125°)$ and $z_2 = 4(\cos 35° + i \sin 35°)$, find the product $z_1 z_2$ (leave answer in polar form).

$z_1 \cdot z_2 = 12 \cdot 4\left(\cos(125° + 35°) + i\sin(125° + 35°)\right)$

$= 48(\cos 160° + i\sin 160°)$

5. Calculate $(-5\sqrt{3}+5i)^5$ (express final answer in standard form exactly — no decimals).



$r = \sqrt{(-5\sqrt{3})^2 + 5^2} = \sqrt{75+25} = \sqrt{100} = 10$

$\tan\theta = \frac{5}{-5\sqrt{3}} = -\frac{1}{\sqrt{3}}$

$\theta = 150°$

polar form: $\left(10(\cos 150° + i\sin 150°)\right)^5$

$= 10^5\left(\cos(150°\cdot 5) + i\sin(150°\cdot 5)\right)$

$= 100000(\cos 750° + i\sin 750°)$

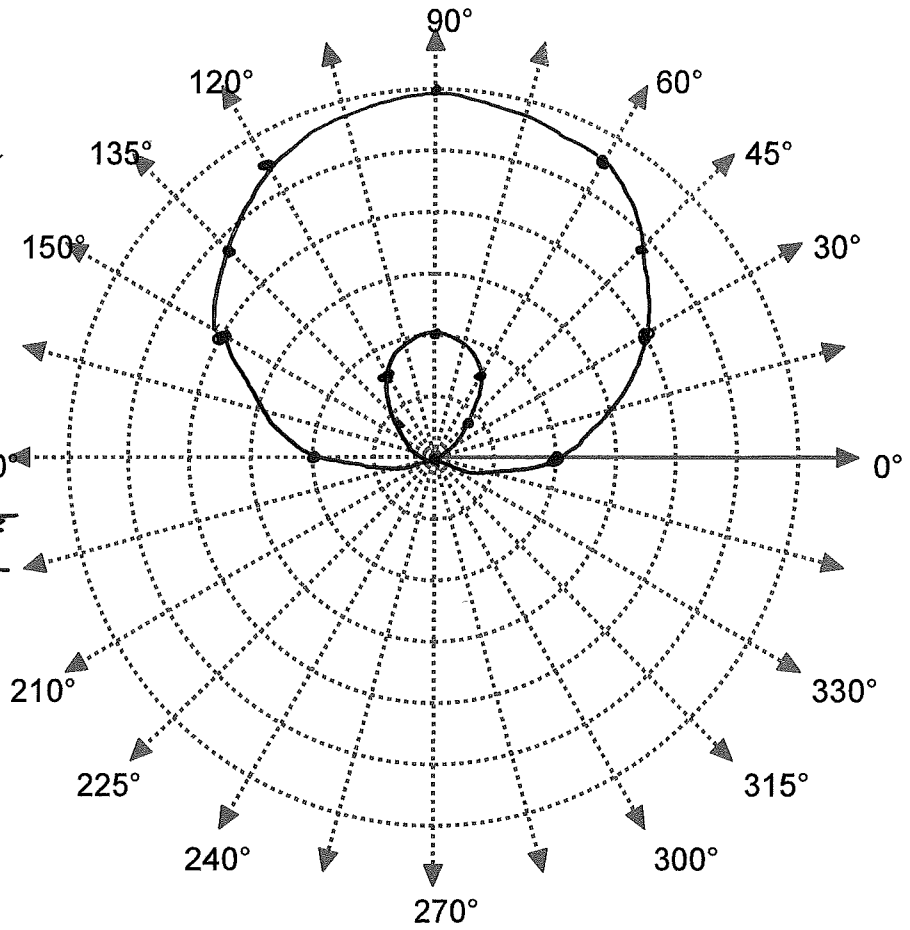
$= 100000(\cos 30° + i\sin 30°)$

$= 100000\left(\frac{\sqrt{3}}{2} + i\frac{1}{2}\right) = \boxed{50000\sqrt{3} + 50000i}$

6. Complete the table below by evaluating the equation: $r = 2+4\sin\theta$.

Plot each of the points, and then connect them in order to draw a sketch of the graph.

θ	r
0°	2
30°	4
45°	$2+2\sqrt{2} \approx 4.8$
60°	$2+2\sqrt{3} \approx 5.5$
90°	6
120°	$2+2\sqrt{3}$
135°	$2+2\sqrt{2}$
150°	4
180°	2
210°	0 ~~$2-2\sqrt{2} \approx -0.8$~~
225°	~~$2-2\sqrt{2} \approx -1.5$~~ −0.8
240°	~~0~~ −1.5
270°	−2
300°	−1.5
315°	−0.8
330°	0
360°	2



7. Given vectors $\mathbf{u} = \langle 3,-2 \rangle$, and $\mathbf{v} = \langle 1,5 \rangle$:

a) find $5\mathbf{u} - 3\mathbf{v} =$

$= 5\langle 3,-2 \rangle - 3\langle 1,5 \rangle$

$= \langle 15,-10 \rangle - \langle 3,15 \rangle$

$= \boxed{\langle 12,-25 \rangle}$

b) find the magnitude of $\mathbf{u}$: $|\mathbf{u}| =$

$|u| = \sqrt{3^2 + (-2)^2} = \sqrt{9+4} = \boxed{\sqrt{13}}$

c) to the nearest tenth of a degree, find the direction of $\mathbf{v}$.

$\tan\theta = \frac{5}{1}$

$\theta = \tan^{-1}(5)$

$\boxed{\theta = 78.7^\circ}$

d) find $\mathbf{u} \bullet \mathbf{v} =$

$u \cdot v = (3)(1) + (-2)(5)$

$= 3 + (-10) = \boxed{-7}$

e) calculate the angle between $\mathbf{u}$ and $\mathbf{v}$ (to the nearest tenth of a degree)

$\cos\theta = \frac{u \cdot v}{|u||v|}$

$\cos\theta = \frac{-7}{\sqrt{13}\sqrt{26}}$

$\theta = \cos^{-1}\left(\frac{-7}{\sqrt{13}\sqrt{26}}\right) = \boxed{112.4^\circ}$

$|u| = \sqrt{13}$ from b) above

$|v| = \sqrt{1^2+5^2} = \sqrt{1+25} = \sqrt{26}$

$u \cdot v = -7$ from d) above