

Student Name: KEY

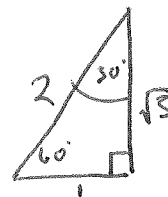
Show all relevant work (use back of pages for scratch paper, if needed). **CIRCLE FINAL ANSWERS.**
Each problem is worth 8 points (and you get one point for clearly writing your name above).

1. Find the exact value of each expression (show answers as fractions not decimals).

$$a) \tan\left(-\frac{\pi}{12}\right) = \tan(-15^\circ) = \tan(30^\circ - 45^\circ) = \frac{\tan 30^\circ - \tan 45^\circ}{1 + \tan 30^\circ \tan 45^\circ}$$

$$= \frac{\frac{1}{\sqrt{3}} - 1}{1 + \frac{1}{\sqrt{3}}(1)} = \frac{\frac{1}{\sqrt{3}} - \frac{\sqrt{3}}{\sqrt{3}}}{\frac{\sqrt{3}}{\sqrt{3}} + \frac{1}{\sqrt{3}}} = \frac{\frac{1-\sqrt{3}}{\sqrt{3}}}{\frac{1+\sqrt{3}}{\sqrt{3}}} = \frac{1-\sqrt{3}}{1+\sqrt{3}} \cdot \frac{1-\sqrt{3}}{1-\sqrt{3}} = \frac{1-\sqrt{3} \cdot \sqrt{3} + \sqrt{3}}{1-\sqrt{3}+\sqrt{3}-3}$$

$$= \frac{4-2\sqrt{3}}{-2} = -2 + \sqrt{3} = \boxed{\sqrt{3} - 2}$$



b) $\sin 105^\circ$

$$\sin(45^\circ + 60^\circ) = \sin 45^\circ \cos 60^\circ + \cos 45^\circ \sin 60^\circ$$

$$\left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$$

$$\frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4} = \boxed{\frac{\sqrt{2} + \sqrt{6}}{4}}$$

2. Write out the Addition Formula for cosine:

$$\cos(\theta + \phi) = \cos\theta \cos\phi - \sin\theta \sin\phi$$

3. Write out the Double-Angle Formula for tangent:

$$\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

4. Circle any of the following expressions (possibly more than one) that is equivalent to $\sin \theta$

$$\cos\left(\frac{\pi}{2} - \theta\right)$$

$$\frac{1}{\csc \theta}$$

$$\sec(-\theta)$$

$$\cos^2 \theta$$

5. Evaluate $\cos(\theta - \phi)$ given that $\tan \theta = \frac{4}{3}$, θ in Quadrant III, $\sin \phi = \frac{-\sqrt{33}}{7}$, ϕ in Quadrant IV.

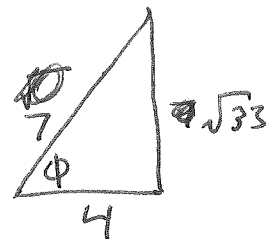
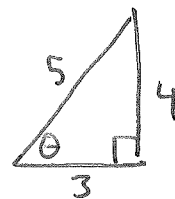
$$\cos(\theta - \phi) = \cos \theta \cos \phi + \sin \theta \sin \phi$$

$$\left(-\frac{3}{5}\right)\left(\frac{4}{7}\right) + \left(-\frac{4}{5}\right)\left(\frac{-\sqrt{33}}{7}\right)$$

$$\frac{-12}{35} + \frac{4\sqrt{33}}{35} =$$

~~$$\frac{-12 + 4\sqrt{33}}{35}$$~~

$$\frac{4\sqrt{33} - 12}{35}$$



6. Find the exact value (fractions not decimals) of: $\cos\left(2 \tan^{-1} \frac{12}{5}\right)$

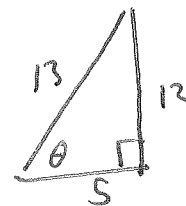
$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta$$

$$(\cos \theta)^2 - (\sin \theta)^2$$

$$\left(\frac{5}{13}\right)^2 - \left(\frac{12}{13}\right)^2$$

$$\frac{25}{169} - \frac{144}{169}$$

$$\frac{-119}{169}$$



7. Circle the one expression below that is equal to: $\tan\theta + \cot\theta = ??$

A. $\cos\theta + \sin\theta$

B. $\sec\theta \csc\theta$

C. $\sec^2\theta$

D. $2 + \sin\theta$

$$\begin{aligned} \tan\theta + \cot\theta &= \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} = \frac{\sin^2\theta}{\cos\theta\sin\theta} + \frac{\cos^2\theta}{\cos\theta\sin\theta} = \frac{\sin^2\theta + \cos^2\theta}{\cos\theta\sin\theta} = \frac{1}{\cos\theta\sin\theta} \\ &= \left(\frac{1}{\cos\theta}\right)\left(\frac{1}{\sin\theta}\right) = \sec\theta \csc\theta \end{aligned}$$

8. Circle the one expression below that is equal to: $\sin^2\theta \cot^2\theta + \cos^2\theta \tan^2\theta = ??$

A. $\cos 2\theta$

B. $\csc^2\theta$

C. 1

D. $2\sin\theta$

$$\sin^2\theta \cot^2\theta + \cos^2\theta \tan^2\theta = \left(\frac{\sin^2\theta}{1}\right)\left(\frac{\cos^2\theta}{\sin^2\theta}\right) + \left(\frac{\cos^2\theta}{1}\right)\left(\frac{\sin^2\theta}{\cos^2\theta}\right) = \cos^2\theta + \sin^2\theta = 1$$

9. Circle the one expression below that is equal to: $1 + \tan\theta \tan\frac{\theta}{2} = ??$

A. $\csc\theta$

B. $\cos\theta + 1$

C. $\cot^2\theta$

D. $\sec\theta$

$$\begin{aligned} 1 + \tan\theta \tan\frac{\theta}{2} &= 1 + \left(\frac{\sin\theta}{\cos\theta}\right)\left(\frac{1-\cos\theta}{\sin\theta}\right) = 1 + \frac{1-\cos\theta}{\cos\theta} = 1 + \frac{1}{\cos\theta} - \frac{\cos\theta}{\cos\theta} \\ &= 1 + \sec\theta - 1 = \sec\theta \end{aligned}$$

10. Find all solutions for θ in the given equations (answers may be in either degrees or radians).

a) $\sin(3\theta) - 2\sin^2(3\theta) = 0$

$$(\sin 3\theta)(1 - 2\sin 3\theta) = 0$$

$$\sin 3\theta = 0 \quad \text{or} \quad 1 - 2\sin 3\theta = 0$$

$$-2\sin 3\theta = -1$$

$$\sin 3\theta = 1/2$$

$$3\theta = 0^\circ + 360^\circ k, 180^\circ + 360^\circ k, \quad 3\theta = 30^\circ + 360^\circ k, 150^\circ + 360^\circ k$$

$$\theta = 0^\circ + 120^\circ k, 60^\circ + 120^\circ k, 10^\circ + 120^\circ k, 50^\circ + 120^\circ k$$

b) $\tan \theta = 7$ (hint: use calculator and express answer to two decimal places)

$$\theta = \tan^{-1}(7)$$

$$\theta = 81.87^\circ + 180^\circ k$$

c) $\cos 2\theta = 3\sin \theta - 1$

$$1 - 2\sin^2 \theta = 3\sin \theta - 1$$

$$0 = 2\sin^2 \theta + 3\sin \theta - 2$$

$$(2\sin \theta - 1)(\sin \theta + 2) = 0$$

$$2\sin \theta - 1 = 0$$

$$2\sin \theta = 1$$

$$\sin \theta = 1/2$$

$$\text{or} \quad \sin \theta + 2 = 0$$

$$\sin \theta = -2$$

Never

$$\theta = 30^\circ + 360^\circ k, 150^\circ + 360^\circ k$$