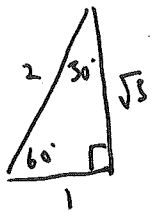
MATH-1150 PRACTICE Exam #4 Instructor: Eric Harshbarger

Student Name: KEY

Show all relevant work (use back of pages for scratch paper, if needed). **CIRCLE FINAL ANSWERS.** Each problem is worth 8 points (and you get one point for clearly writing your name above).

1. Find the exact value of each expression (show answers as fractions not decimals).

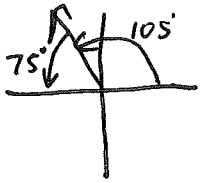
a) $\tan\left(-\frac{\pi}{12}\right) = \tan(-15^\circ) = \tan(30^\circ - 45^\circ) = \frac{\tan 30^\circ - \tan 45^\circ}{1 + \tan 30^\circ \tan 45^\circ}$



$= \frac{\frac{1}{\sqrt{3}} - 1}{1 + \frac{1}{\sqrt{3}}(1)} = \frac{\frac{1}{\sqrt{3}} - \frac{\sqrt{3}}{\sqrt{3}}}{\frac{\sqrt{3}}{\sqrt{3}} + \frac{1}{\sqrt{3}}} = \frac{\frac{1-\sqrt{3}}{\sqrt{3}}}{\frac{\sqrt{3}+1}{\sqrt{3}}} = \left(\frac{1-\sqrt{3}}{\sqrt{3}}\right)\left(\frac{\sqrt{3}}{1+\sqrt{3}}\right) = \frac{1-\sqrt{3}}{1+\sqrt{3}}$

$= \frac{1-\sqrt{3}}{1+\sqrt{3}} \cdot \frac{1-\sqrt{3}}{1-\sqrt{3}} = \frac{1-\sqrt{3}-\sqrt{3}+3}{1-\sqrt{3}+\sqrt{3}-3} = \frac{4-2\sqrt{3}}{-2} = \frac{4}{-2} - \frac{2\sqrt{3}}{-2} = \boxed{\sqrt{3}-2}$

b) $\sin 105^\circ = \boxed{\frac{\sqrt{6}+\sqrt{2}}{4}}$



$\bar{\theta} = 75^\circ$

$\sin 75^\circ = \sin(45^\circ + 30^\circ) = \sin 45^\circ \cos 30^\circ + \cos 45^\circ \sin 30^\circ$

$\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$

$\frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \frac{\sqrt{6}+\sqrt{2}}{4}$

2. Write out the Addition Formula for cosine:

$\cos(\theta+\phi) =$ $\cos\theta\cos\phi - \sin\theta\sin\phi$

3. Write out the Double-Angle Formula for tangent:

$\tan 2\theta =$ $\frac{2\tan\theta}{1-\tan^2\theta}$

4. Circle any of the following expressions (possibly more than one) that is equivalent to $\sin\theta$

$\cos\left(\frac{\pi}{2}-\theta\right)$ $\quad$ $\frac{1}{\csc\theta}$ $\quad$ $\sin(-\theta)$ $\quad$ $\cos^2\theta$

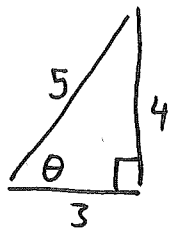
5. Evaluate $\cos(\theta-\phi)$ given that $\tan\theta=\frac{4}{3}$, θ in Quadrant III, $\sin\phi=\frac{-\sqrt{33}}{7}$, ϕ in Quadrant IV.

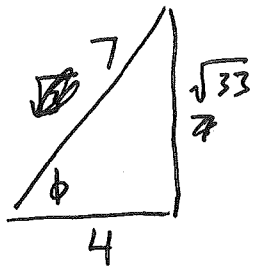
$\cos(\theta-\phi) = \cos\theta\cos\phi + \sin\theta\sin\phi$

$\left(\frac{-3}{5}\right)\left(\frac{4}{7}\right) + \left(\frac{-4}{5}\right)\left(\frac{-\sqrt{33}}{7}\right)$

$\frac{-12+4\sqrt{33}}{35}$

$\boxed{\frac{4\sqrt{33}-12}{35}}$





6. Find the exact value (fractions not decimals) of: $\cos\left(2\tan^{-1}\frac{12}{5}\right)$

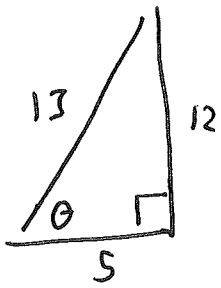
θ

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$\left(\frac{5}{13}\right)^2 - \left(\frac{12}{13}\right)^2$

$\frac{25}{169} - \frac{144}{169}$

$\boxed{\frac{-119}{169}}$



7. Find all solutions for θ in the given equations (answers may be in either degrees or radians).

a) $\sin(3\theta) - 2\sin^2(3\theta) = 0$

$(\sin 3\theta)(1 - 2\sin 3\theta) = 0$

$\sin 3\theta = 0$ or $1 - 2\sin 3\theta = 0 \Rightarrow -2\sin 3\theta = -1$, $\sin 3\theta = \frac{1}{2}$

$3\theta = 0° + 360°k,\ 180° + 360°k,\ 30° + 360°k,\ 150° + 360°k$

$\theta = 0° + 120°k,\ 60° + 120°k,\ 10° + 120°k,\ 50° + 120°k$

b) $\tan\theta = 7$

$\theta = \tan^{-1}(7)$

$\theta = 81.87° + 180°k$

c) $\cos 2\theta = 3\sin\theta - 1$

$1 - 2\sin^2\theta = 3\sin\theta - 1$

$0 = 2\sin^2\theta + 3\sin\theta - 2$

$(2\sin\theta - 1)(\sin\theta + 2) = 0$

$2\sin\theta - 1 = 0$ or $\sin\theta + 2 = 0$

$2\sin\theta = 1$ $\sin\theta = -2$ ✗

$\sin\theta = \frac{1}{2}$

$\theta = 30° + 360°k,\ 150° + 360°k$

8. Circle the one expression below that is equal to: $\tan\theta + \cot\theta = ??$

A. $\cos\theta + \sin\theta$

B. $\sec\theta \csc\theta$

C. $\sec^2\theta$

D. $2 + \sin\theta$

$\tan\theta + \cot\theta = \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} = \frac{\sin^2\theta}{\cos\theta\sin\theta} + \frac{\cos^2\theta}{\cos\theta\sin\theta} = \frac{\sin^2\theta + \cos^2\theta}{\cos\theta\sin\theta} =$

$\frac{1}{\cos\theta\sin\theta} = \left(\frac{1}{\cos\theta}\right)\left(\frac{1}{\sin\theta}\right) = \sec\theta\csc\theta$

9. Circle the one expression below that is equal to: $\sin^2\theta\cot^2\theta + \cos^2\theta\tan^2\theta = ??$

A. $\cos 2\theta$

B. $\csc^2\theta$

C. 1

D. $2\sin\theta$

$\sin^2\theta\cot^2\theta + \cos^2\theta\tan^2\theta = \left(\frac{\sin^2\theta}{1}\right)\left(\frac{\cos^2\theta}{\sin^2\theta}\right) + \left(\frac{\cos^2\theta}{1}\right)\left(\frac{\sin^2\theta}{\cos^2\theta}\right)$

$= \cos^2\theta + \sin^2\theta = 1$

10. Circle the one expression below that is equal to: $1 + \tan\theta\tan\frac{\theta}{2} = ??$

A. $\csc\theta$

B. $\cos\theta + 1$

C. $\cot^2\theta$

D. $\sec\theta$

$1 + \tan\theta\tan\frac{\theta}{2} = 1 + \left(\frac{\sin\theta}{\cos\theta}\right)\left(\frac{1-\cos\theta}{\sin\theta}\right) = 1 + \frac{1-\cos\theta}{\cos\theta} = 1 + \frac{1}{\cos\theta} - \frac{\cos\theta}{\cos\theta}$

$= 1 + \sec\theta - 1 = \sec\theta$