MATH-1150 PRACTICE EXAM #3 Instructor: Eric Harshbarger

Student Name: KEY

Show all relevant work (use back of pages for scratch paper, if needed). **CIRCLE FINAL ANSWERS.**

1. [5 pts each] Evaluate the following and express answers in standard form $a + bi$:

a) $(2 - 4i) - (5 + 8i) = \boxed{-3 - 12i}$

b) $\frac{7-3i}{4+2i} \cdot \frac{4-2i}{4-2i} = \frac{28-14i-12i+6i^2}{16-8i+8i-4i^2} = \frac{22-26i}{20} = \frac{22}{20} - \frac{26i}{20}$

$= \boxed{\frac{11}{10} - \frac{13}{10}i}$

c) $i^{62} = i^{60} \cdot i^2 = (i^4)^{15} \cdot (-1) = 1^{15} \cdot (-1) = 1 \cdot (-1) = \boxed{-1}$

2. [5 pts] Find all solutions of the equation $3x^2+2x+7=0$ and express them in the form $a + bi$.

$a = 3$
$b = 2$
$c = 7$

$$x = \frac{-2 \pm \sqrt{2^2 - 4(3)(7)}}{2(3)} = \frac{-2 \pm \sqrt{4-84}}{6} = \frac{-2 \pm \sqrt{-80}}{6}$$

$$= \frac{-2 \pm 2i\sqrt{80}}{6} = \frac{-2 \pm i\sqrt{16}\sqrt{5}}{6} = \frac{-2 \pm 4i\sqrt{5}}{6} = \frac{-2}{6} \pm \frac{4i\sqrt{5}}{6}$$

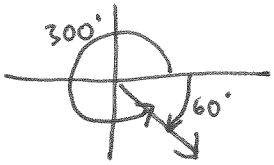
$$= \boxed{\frac{-1}{3} \pm \frac{2i\sqrt{5}}{3}}$$

3. [30 pts] Complete the table with the exact values (write answers as fractions, *not* decimals) of the trigonometric functions. If a value is undefined, write "undef."

θ in degrees	sin θ	cos θ	tan θ	csc θ	sec θ	cot θ
0°	0	1	0	undef	1	undef
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$	2	$\frac{2}{\sqrt{3}}$	$\sqrt{3}$
45°	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	$\sqrt{2}$	$\sqrt{2}$	1
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{2}{\sqrt{3}}$	2	$\frac{1}{\sqrt{3}}$
90°	1	0	undef	1	undef	0

4. [5 pts each] Find the exact (fractions not decimals) value of

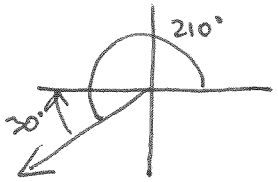
(a) $\sin\frac{5\pi}{3} = \sin 300^\circ = \boxed{-\frac{\sqrt{3}}{2}}$



$\bar{\theta} = 60^\circ$

$\sin 60^\circ = \frac{\sqrt{3}}{2}$

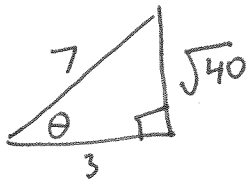
(b) $\sec(210°) = \frac{1}{\cos 210^\circ} = \frac{1}{-\frac{\sqrt{3}}{2}} = \boxed{-\frac{2}{\sqrt{3}}}$



$\bar{\theta} = 30^\circ$

$\cos 30^\circ = \frac{\sqrt{3}}{2}$

5. [5 pts] Find the exact value (fractions not decimals) of $\tan(\underbrace{\cos^{-1}(\frac{3}{7})}_{\theta}) = \boxed{\frac{\sqrt{40}}{3} = \frac{2\sqrt{10}}{3}}$



$7^2 = 3^2 + b^2$

$49 = 9 + b^2$

$40 = b^2$

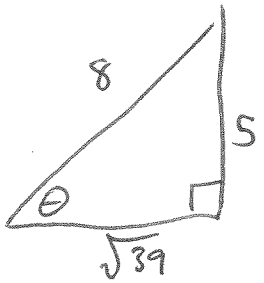
$b = \sqrt{40}$

6. [10 pts] Given θ in Quadrant III and the value of $\sin\theta$, find the exact values (fractions not decimals) of the remaining trigonometric functions:

$\sin\theta = -\frac{5}{8}$ $\qquad$ $\csc\theta = -8/5$

$\cos\theta = -\frac{\sqrt{39}}{8}$ $\qquad$ $\sec\theta = -\frac{8}{\sqrt{39}}$

$\tan\theta = \frac{5}{\sqrt{39}}$ $\qquad$ $\cot\theta = \frac{\sqrt{39}}{5}$



$8^2 = 5^2 + a^2$

$64 = 25 + a^2$

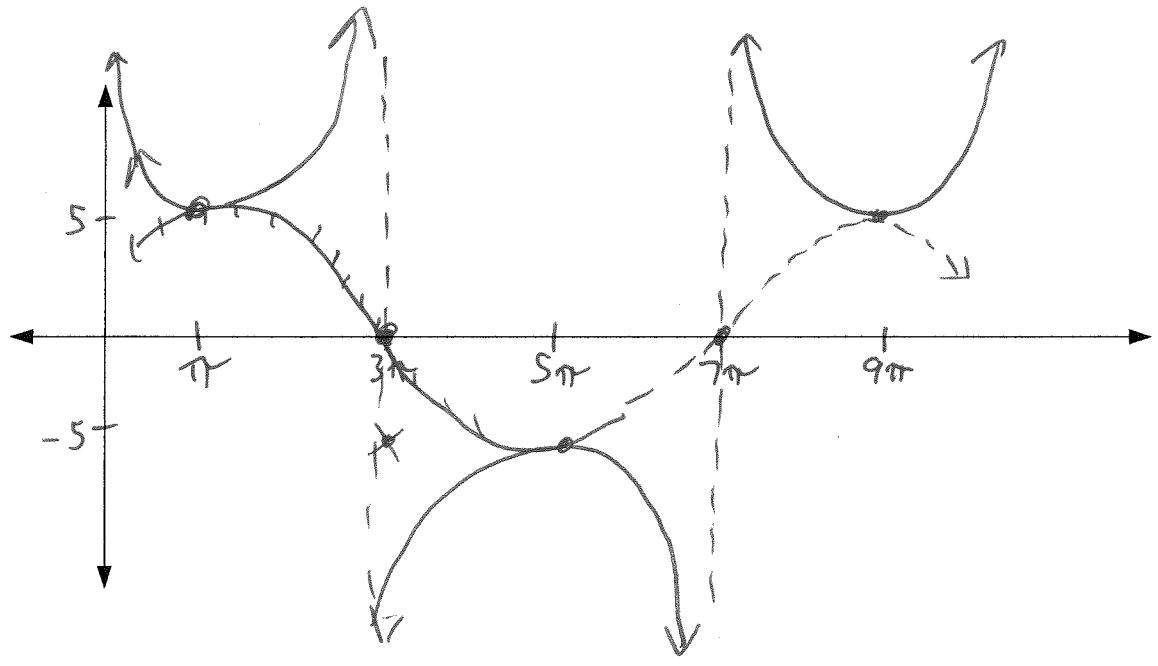
$a^2 = 39$

$a = \sqrt{39}$

$5\cos\frac{1}{4}(x-\pi)$

7. [10 pts] Find the period and phase shift of the function $y = 5\sec\frac{1}{4}(x-\pi)$. Then graph one period and label all relevant points along the axes.

Period = $\frac{2\pi}{\frac{1}{4}} = (2\pi)(4) = 8\pi$ $\qquad$ Phase Shift = π to right



8. [10 pts] Given the following information, solve for all possible triangles. If no triangle is possible, write, "Impossible." If two triangles are possible, list both sets of values. Express answers to one decimal place.

SSS
Law of Cosines

$\angle A =$ 26.4°

$\angle B =$ 117.3°

$\angle C =$ 36.3°

$a = 3.0$

$b = 6.0$

$c = 4.0$

$3^2 = 6^2 + 4^2 - 2(6)(4)\cos A$

$9 = 36 + 16 - 48\cos A$

$9 = 52 - 48\cos A$

$-43 = -48\cos A$

$\frac{43}{48} = \cos A$

$A = \cos^{-1}\left(\frac{43}{48}\right)$

$A = 26.4°$

$6^2 = 3^2 + 4^2 - 2(3)(4)\cos B$

$36 = 9 + 16 - 24\cos B$

$36 = 25 - 24\cos B$

$11 = -24\cos B$

$\frac{-11}{24} = \cos B$

$B = \cos^{-1}\left(\frac{-11}{24}\right)$

$B = 117.3°$

$4^2 = 3^2 + 6^2 - 2(3)(6)\cos C$

$16 = 9 + 36 - 36\cos C$

$16 = 45 - 36\cos C$

$-29 = -36\cos C$

$\frac{29}{36} = \cos C$

$C = \cos^{-1}\left(\frac{29}{36}\right)$

$C = 36.3°$

9. [10 pts] A 300 foot support cable is attached to the top of a 70 foot flagpole. The flagpole itself is mounted at the top of a hill that has an angle of inclination of 38°. How far down the side of hill will the cable reach, as measured from the base of the pole (distance d in the figure)?

$\frac{\sin 128°}{300} = \frac{\sin\theta}{70}$

$\sin\theta = \frac{70\sin 128°}{300}$

$\theta = \sin^{-1}\left(\frac{70\sin 128°}{300}\right)$

$\theta = 10.6°$

$\frac{\sin 128°}{300} = \frac{\sin 41.4°}{d}$

$d = \frac{300\sin 41.4°}{\sin 128°}$

$d = 251.8'$