MATH-1150 PRACTICE Exam #2 Instructor: Eric Harshbarger

Student Name: KEY

Show all relevant work (use back of pages for scratch paper, if needed). **CIRCLE FINAL ANSWERS.** Each problem is worth 7 points.

1. Evaluate each expression (do not use a calculator or decimal places):

a) $\log_6 18 + \log_6 12$ $= \log_6 (18)(12) = \log_6 216 = \boxed{3}$

b) $\ln e^4$ $= \log_e e^4 = \boxed{4}$

2. Evaluate: $\log_{13} 1278$ (use calculator to four decimal places).

$\log_{13} 1278 = \dfrac{\log 1278}{\log 13} = \boxed{2.7888}$

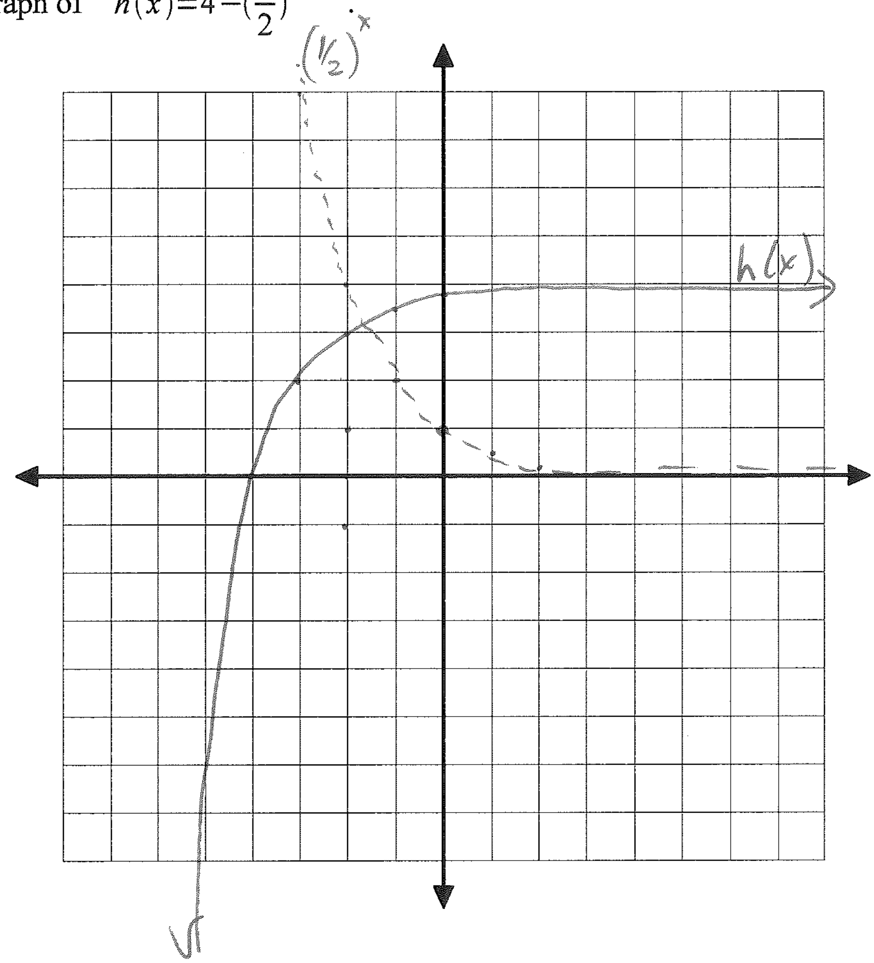
3. Combine each expression into a single logarithm and simplify, if possible:

a) $\log_4(x^2 - 5x + 6)$ - $\log_4(x - 3)$ $= \log_4 \left(\dfrac{x^2 - 5x + 6}{x - 3}\right) = \log_4 \dfrac{(x-2)\cancel{(x-3)}}{\cancel{(x-3)}}$

$= \boxed{\log_4 (x-2)}$

b) $\ln 2 + \ln(x+1) - 4\ln(3x+7)$ $= \boxed{\ln \dfrac{2(x+1)}{(3x+7)^4}}$

4. Sketch the graph of $h(x)=4-(\frac{1}{2})^{x+2}$.



Transformations
- left 2
- flip vertically
- up 4

5. A sum of \$1000 is invested at an interest rate of 7% per year. Find the time (in years, to two decimal places) required for the money to triple if the interest is compounded:

$P = 1000$
$r = 7\% = 0.07$
$A(t) = 3000$
$t = ?$

a) quarterly (that is, four times per year).

$n = 4$

$3000 = 1000\left(1+\frac{0.07}{4}\right)^{4t}$

$3 = (1.0175)^{4t}$

$\log 3 = \log(1.0175)^{4t}$

$\log 3 = 4t \log 1.0175$

$t = \frac{\log 3}{4 \log 1.0175} = \boxed{15.83 \text{ yrs}}$

b) continuously

$3000 = 1000e^{0.07t}$

$3 = e^{0.07t}$

$\ln 3 = 0.07t$

$t = \frac{\ln 3}{0.07} \approx \boxed{15.69 \text{ yrs}}$

6. Find the solution to each equation; you may leave the answer in exact form, or rounded to three decimal places:

a) $3^{x+2}=17$

$\log 3^{x+2} = \log 17$

$(x+2)\log 3 = \log 17$

$x+2 = \frac{\log 17}{\log 3}$

$x = \frac{\log 17}{\log 3} - 2 \approx \boxed{0.579}$

b) $e^{3-5x}=16$

$\ln e^{3-5x} = \ln 16$

$3-5x = \ln 16$

$-5x = \ln 16 - 3$

$x = \frac{\ln 16 - 3}{-5} = \frac{3-\ln 16}{5} \approx \boxed{0.045}$

c) $2\log_5(3x-4)=6$

$\log_5(3x-4) = 3$

$5^3 = 3x-4$

$125 = 3x-4$

$129 = 3x$

$\boxed{x = 43}$

d) $x^2 10^x - x10^x = 2(10^x)$

$x^2 10^x - x\,10^x - 2(10^x) = 0$

$10^x(x^2-x-2) = 0$

$10^x(x-2)(x+1) = 0$

$10^x = 0$ $\quad x-2=0 \quad x+1=0$

$\times$ $\quad \boxed{x=2 \qquad x=-1}$

7. Iodine-135 is a radioactive element with a half-life of 8 days. If you currently have a 320 mg sample of this substance, how long will it be before it has decayed down to ~~75~~mg?

60

$h = 8$ days

$m_0 = 320$

$m(t) = \cancel{75}\ 60$

$\cancel{m(t) = m_0 \cdot \left(\tfrac{1}{2}\right)^{t/h}}$

$\cancel{75 = 320 \cdot (0.5)^{t/8}}$

$\cancel{0.234375 = 0.5^{t/8}}$

$\cancel{\log(0.234375) = \log(0.5)^{t/8}}$

$\cancel{\log(0.234375) = \tfrac{t}{8}\log(0.5)}$

$\cancel{t = \dfrac{8\log(0.234375)}{\log(0.5)}} = \cancel{16.75}$ days.

$$m(t) = m_0 \cdot \left(\tfrac{1}{2}\right)^{t/h}$$

$$60 = 320 \cdot (0.5)^{t/8}$$

$$\frac{60}{320} = 0.5^{t/8}$$

$$\log\left(\tfrac{3}{16}\right) = \log(0.5)^{t/8}$$

$$\log(0.1875) = \tfrac{t}{8}\log(0.5)$$

$$\boxed{19.32 \text{ days}} \approx \frac{8\log(0.1875)}{\log(0.5)} = t$$

8. The initial population of a colony of rabbits is 106, and it is known to double every 17 days.

a) How many rabbits would you expect to be in the colony after 80 days?

$n_0 = 106$

$a = 17$

$$n(t) = n_0 \cdot 2^{t/a}$$

$$n(80) = 106 \cdot 2^{80/17} = \boxed{2766 \text{ rabbits}}$$

b) Calculate the relative growth rate, *r*, of the colony.

$$n(t) = n_0 e^{rt}$$

$$212 = 106e^{r17}$$

$$2 = e^{17r}$$

$$\ln 2 = 17r$$

$$r = \frac{\ln 2}{17} \approx \cancel{0.40}\ 0.0408 = \boxed{4.08\%}$$