MATH-1150 PRACTICE Exam #2 Instructor: Eric Harshbarger

Student Name: KEY

Show all relevant work (use back of pages for scratch paper, if needed). **CIRCLE FINAL ANSWERS**. Each problem is worth 7 points.

1. Evaluate each expression (do not use a calculator or decimal places):

a) $\log_6 18 + \log_6 12$ $= \log_6(18 \cdot 12) = \log_6(216) = \boxed{3}$

b) $\ln e^4$ $= \boxed{4}$

2. Evaluate: $\log_{13} 1278$ (use calculator to four decimal places).

$\log_{13} 1278 = \dfrac{\log 1278}{\log 13} = 2.7888$

3. Combine each expression into a single logarithm and simplify, if possible:

a) $\log_4(x^2 - 5x + 6) - \log_4(x - 3)$ $= \log_4\left(\dfrac{x^2-5x+6}{x-3}\right) = \log_4 \dfrac{\cancel{(x-3)}(x-2)}{\cancel{(x-3)}} =$

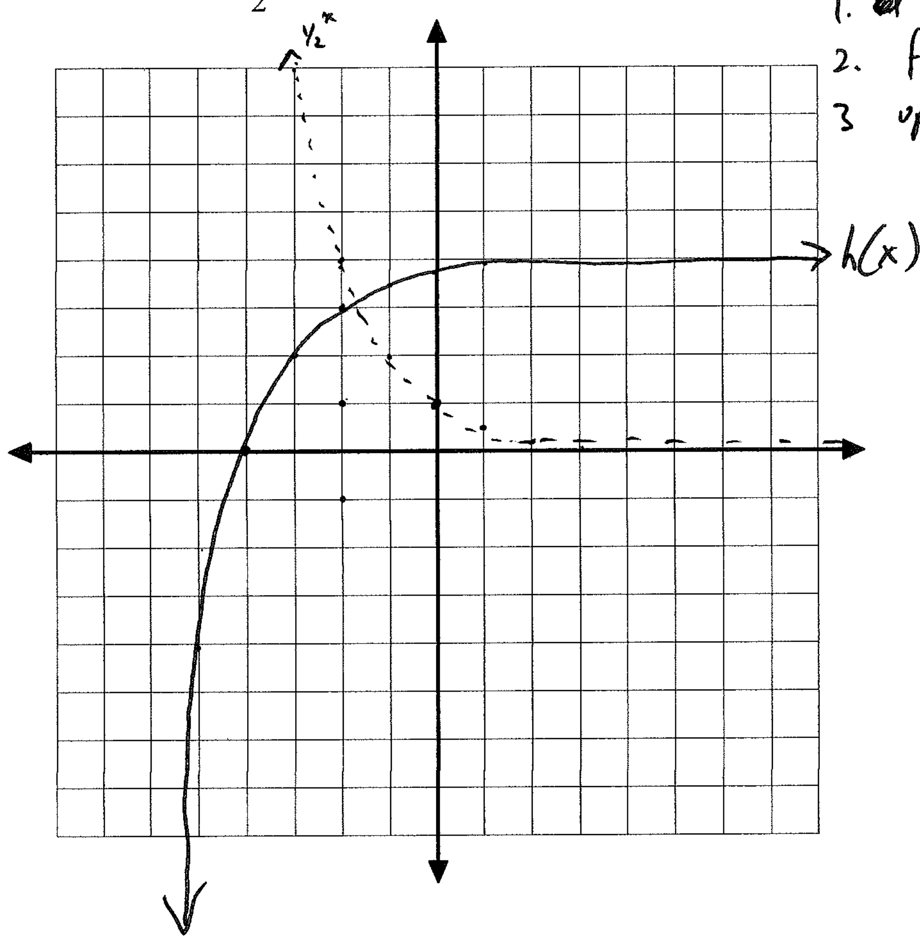
$\boxed{\log_4(x-2)}$

b) $\ln 2 + \ln(x+1) - 4\ln(3x+7)$ $= \ln 2 + \ln(x+1) - \ln(3x+7)^4 =$

$\ln\left(\dfrac{2(x+1)}{(3x+7)^4}\right)$

4. Sketch the graph of $h(x)=4-(\frac{1}{2})^{x+2}$.

1. left 2
2. flip vertically
3 up 4



5. A sum of \$1000 is invested at an interest rate of 7% per year. Find the time (in years) required for the money to triple if the interest is compounded:

$P=1000$
$r=7\%=0.07$
$A(t)=3000$
$t=?$

$n=4$

a) quarterly (that is, four times per year).

$$3000 = 1000\left(1+\frac{0.07}{4}\right)^{4t}$$
$$3 = (1.0175)^{4t}$$
$$\log 3 = 4t\log(1.0175)$$
$$t = \frac{\log 3}{4\log(1.0175)} = \boxed{15.83 \text{ years}}$$

b) continuously

$$3000 = 1000e^{0.07t}$$
$$3 = e^{0.07t}$$
$$\ln 3 = 0.07t$$
$$t = \frac{\ln 3}{0.07} \approx \boxed{15.69 \text{ years}}$$

6. Find the solution to each equation; you may leave the answer in exact form, or rounded to three decimal places:

a) $3^{x+2}=17$

$\log 3^{x+2} = \log 17$

$(x+2)\log 3 = \log 17$

$x+2 = \frac{\log 17}{\log 3}$

$x = \frac{\log 17}{\log 3} - 2 = \boxed{0.579}$

b) $e^{3-5x}=16$

$\ln e^{3-5x} = \ln 16$

$3-5x = \ln 16$

$-5x = \ln 16 - 3$

$x = \frac{\ln 16 - 3}{-5} = \frac{3-\ln 16}{5} \approx \boxed{0.045}$

c) $2\log_5(3x-4)=6$

$\log_5(3x-4) = 3$

$5^3 = 3x-4$

$125 = 3x-4$

$3x = 129$

$\boxed{x = 43}$

d) $x^2 10^x - x10^x = 2(10^x)$

$x^2 10^x - x10^x - 2(10^x) = 0$

$10^x(x^2 - x - 2) = 0$

$10^x(x+1)(x-2) = 0$

$10^x = 0$ never

$x+1=0$ $\boxed{x=-1}$

$x-2=0$ $\boxed{x=2}$

7. A wooden artifact discovered at a burial ground contains 82% of the carbon-14 that is present in living trees. How long ago was the artifact made? (The half-life of carbon-14 is 5730 years).

$m(t) = 0.82\, m_0$

$h = 5730$ yrs

$t = ?$

$m(t) = m_0 \cdot 2^{-t/h}$

$0.82 \cancel{m_0} = \cancel{m_0} \cdot 2^{-t/5730}$

$0.82 = 2^{-t/5730}$

$\log 0.82 = \frac{-t}{5730} \log 2$

$5730 \log 0.82 = -t \log 2$

$\boxed{1640.5 \text{ years}} \approx -\frac{5730 \log 0.82}{\log 2} = t$

8. The initial population of a colony of rabbits is 106, and it is known to double every 17 days.

a) How many rabbits would you expect to be in the colony after 230 days?

$n_0 = 106$

$a = 17$

$n(t) = 106 \cdot 2^{t/17}$

$n(230) = 106 \cdot 2^{230/17} = \boxed{1,253,327 \text{ rabbits}}$

b) Calculate the relative growth rate, *r*, of the colony.

at 17 days there will be twice as many rabbits as initially; 212

$n(t) = 106 e^{rt}$

$n(17) = 212 = 106 e^{r17}$

$2 = e^{17r}$

$\ln 2 = \ln e^{17r}$

$\ln 2 = 17r$

$r = \frac{\ln 2}{17} = 0.0408 \approx \boxed{r = 4.08\%}$