MATH-1150 PRACTICE Exam #2 Instructor: Eric Harshbarger

Student Name: KEY

Show all relevant work (use back of pages for scratch paper, if needed). **CIRCLE FINAL ANSWERS.**

1. [7 pts each] Evaluate each expression (do not use a calculator or decimal places):

a) $\log_6 18 + \log_6 12 = \log_6(18 \cdot 12) = \log_6(216) = \boxed{3}$

b) $\log_7 30 - \log_7 14 - \log_7 105 = \log_7\left(\frac{30}{14 \cdot 105}\right) = \log_7\left(\frac{30}{1470}\right) =$

$\log_7 \frac{1}{49} = \log_7(7^{-2}) = \boxed{-2}$

2. [7 pts] Use a calculator to evaluate $\log_{13} 1278$ rounded to four decimal places:

$\log_{13} 1278 = \frac{\log 1278}{\log 13} = \boxed{2.7888}$

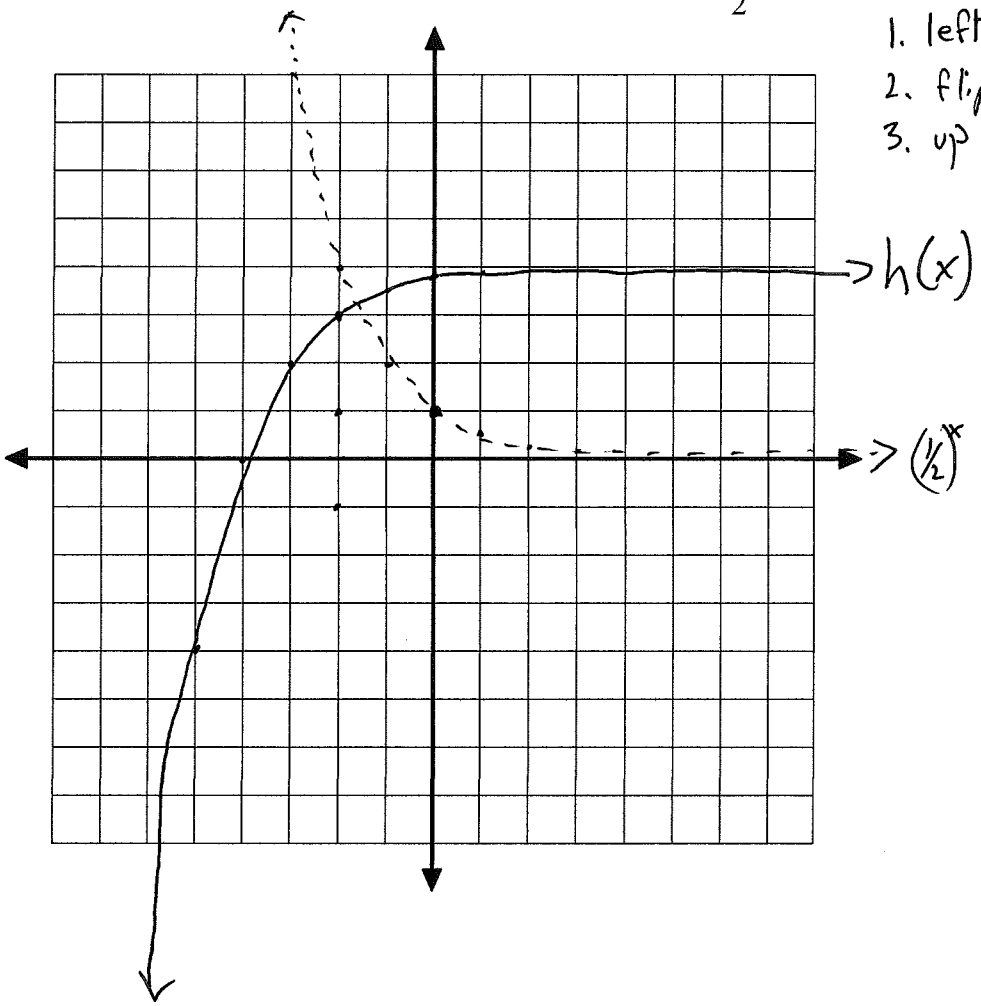
3. [7 pts] Imagine a science fiction scenario in which an outbreak of zombies occurs. Experts from the Center for Disease Control establish the following growth model to predict the population of zombies where t is the number of days since the beginning of outbreak: $z(t) = \frac{8900}{0.01 + 12.7e^{-0.059t}}$. How many zombies would one expect there to be after 30 days?

$z(30) = \frac{8900}{0.01 + 12.7e^{-0.059(30)}} = \boxed{4095 \text{ zombies}}$

4. [7 pts] Combine $\ln 2 + \ln(x+1) - 4\ln(3x+7)$ into a single logarithm and simplify, if possible:

$\ln 2 + \ln(x+1) - \ln(3x+7)^4 = \boxed{\ln\left(\frac{2(x+1)}{(3x+7)^4}\right)}$

5. [7 pts] On the grid provided, sketch and label the graph of $h(x)=4-\left(\frac{1}{2}\right)^{x+2}$



1. left 2
2. flip vertically
3. up 4

6. [7 pts each] A sum of $1000 is invested at an interest rate of 8% per year. Find the time (in years, to two decinal places) required for the money to triple if the interest is compounded:

a) quarterly (that is, four times per year).

$n=4$

$A(t)=P\left(1+\frac{r}{n}\right)^{nt}$

$P=1000$

$r=8\%=0.08$

$t=?$

$A(t)=3000$

$3000=1000\left(1+\frac{0.08}{4}\right)^{4t}$

$3=(1.02)^{4t}$

$\log 3=4t\log(1.02)$

$t=\frac{\log 3}{4\log(1.02)}\approx$ **13.87 years**

b) continuously

$A(t)=Pe^{rt}$

$3000=1000e^{0.08t}$

$3=e^{0.08t}$

$\ln 3=0.08t$

$t=\frac{\ln 3}{0.08}=$ **13.73 years**

7. [14 pts] The initial population of a colony of rabbits is 106, and it is known to double every 47 days.

a) Write a function that models the population of this specific colony of rabbits after t days.

$n_0 = 106$

$a = 47$

$$n(t) = 106 \cdot 2^{t/47}$$

b) How many rabbits would you expect to be in the colony after 230 days?

$$n(230) = 106 \cdot 2^{230/47} = 3150 \text{ rabbits}$$

c) Calculate the relative growth rate, r, of the colony.

$n(t) = n_0 e^{rt}$

$212 = 106\, e^{r(47)}$

$2 = e^{47r} \longrightarrow \ln 2 = 47r$

$r = \frac{\ln 2}{47} = 0.0147 \approx 1.47\%$

8. [7 pts] A wooden artifact discovered at a burial ground contains 82% of the carbon-14 that is present in living trees. How long ago was the artifact made? (The half-life of carbon-14 is 5730 years).

$m(t) = m_0 \cdot (\frac{1}{2})^{t/h}$

$h = 5730$

$0.82 m_0 = m_0 (1/2)^{t/5730}$

$0.82 = (0.5)^{t/5730}$

$\log(0.82) = \frac{t}{5730} \log(0.5)$

$t = \frac{5730 \log(0.82)}{\log(0.5)} = 1640.5 \text{ years}$

9. [7 pts each] Find the solution to each equation; you may leave the answer in exact form, or rounded to three decimal places:

a) $3^{x+2}=17$

$\log 3^{x+2} = \log 17$

$(x+2)\log 3 = \log 17$

$x+2 = \frac{\log 17}{\log 3}$

$x = \frac{\log 17}{\log 3} - 2 \approx \boxed{0.579}$

b) $2\log_5(3x-4)=6$

$\log_5(3x-4) = 3$

$5^3 = 3x-4$

$125 = 3x-4$

$129 = 3x$

$\boxed{x = 43}$

c) $\log_3(x+15) - \log_3(x-1) = 2$

$\log_3 \frac{x+15}{x-1} = 2$

$3^2 = \frac{x+15}{x-1}$

$\frac{9}{1} = \frac{x+15}{x-1}$

$9(x-1) = x+15$

$9x-9 = x+15$

$8x = 24$

$\boxed{x = 3}$

d) $e^{2x}-9e^x+20=0$

$(e^x)^2 - 9e^x + 20 = 0$

$a = e^x$

$a^2 - 9a + 20 = 0$

$(a-4)(a-5) = 0$

$a-4=0$ or $a-5=0$

$a=4$ or $a=5$

$e^x = 4$ or $e^x = 5$

$\boxed{x = \ln 4 \text{ or } x = \ln 5}$

$\boxed{x \approx 1.386 \text{ or } x \approx 1.609}$