MATH-1150 Exam #5 Instructor: Eric Harshbarger

Student Name: KEY

Show all relevant work (use back of pages for scratch paper, if needed). **CIRCLE FINAL ANSWERS.**

1. [5 pts each] Evaluate the following and express answers in standard form *a* + *bi*:

a) $\frac{5+6i}{4-3i} \cdot \frac{4+3i}{4+3i} = \frac{20+15i+24i+18i^2}{16+12i-12i-9i^2} = \frac{20-18+39i}{16+9} = \frac{2+39i}{25} = \boxed{\frac{2}{25}+\frac{39}{25}i}$

b) $(8-3i) - (2-i) = \boxed{6-2i}$

c) $i^{17} = (i^4)^4 \cdot i = 1^4 \cdot i = 1 \cdot i = \boxed{i}$

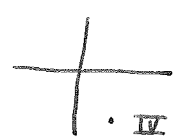
2. [5 pts] Find all solutions of the equation $\underset{a}{6x^2} \underset{b}{-5x} \underset{c}{+3} = 0$ and express them in the form *a* + *bi*.

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x = \frac{5 \pm \sqrt{(-5)^2-4(6)(3)}}{2(6)} = \frac{5 \pm \sqrt{25-72}}{12} = \frac{5 \pm \sqrt{-47}}{12} = \boxed{\frac{5}{12} \pm \frac{\sqrt{47}}{12}i}$

3. [5 pts] Convert rectangular coordinates (5, -8) into polar coordinates ~~(answer to two decimal places)~~.

$r = \sqrt{5^2+(-8)^2} = \sqrt{25+64} = \sqrt{89}$



$\tan\theta = \frac{-8}{5}$

$\tan^{-1}(\tan\theta) = \tan^{-1}\left(\frac{-8}{5}\right)$

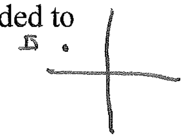
$\theta = -58.0^\circ$

$\theta = 302^\circ$

$\boxed{(\underset{r}{\sqrt{89}}, \underset{\theta}{302^\circ})}$

$(9.43, 302^\circ)$

4. [20 pts.] Calculate $(-\sqrt{3}+i)^7$ (express answer in standard $a+bi$ form, exact answer or rounded to two decimal places).



$r = \sqrt{(-\sqrt{3})^2 + 1^2} = \sqrt{3+1} = \sqrt{4} = 2$

$\tan\theta = \frac{1}{-\sqrt{3}}$

$\bar{\theta} = 30^\circ$

$\theta = 150^\circ$

$\left(2(\cos 150^\circ + i \sin 150^\circ)\right)^7 = 128(\cos 1050^\circ + i \sin 1050^\circ)$

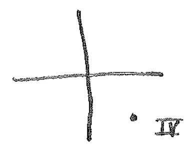
$128(\cos 330^\circ + i \sin 330^\circ)$

$\boxed{110.85 - 64i}$

or $\boxed{64\sqrt{3} - 64i}$

5. [20 pts.] Find the cube roots (that is, the three third roots) of $4\sqrt{2} - 4\sqrt{2}\,i$ (express answers in standard or polar form, exact answers or rounded to two decimal places).

$n = 3$



$r = \sqrt{(4\sqrt{2})^2 + (4\sqrt{2})^2} = \sqrt{32 + 32} = \sqrt{64} = 8$

$\tan\theta = \frac{-4\sqrt{2}}{4\sqrt{2}} = -1$

$\bar{\theta} = 45^\circ$

$\theta = 315^\circ$

$r^{1/n} = 8^{1/3} = 2$

$\frac{\theta}{n} = \frac{315^\circ}{3} = 105^\circ$

$\frac{360^\circ}{n} = \frac{360^\circ}{3} = 120^\circ$

$w_0 = 2(\cos 105^\circ + i \sin 105^\circ) = -0.52 + 1.93i$

$w_1 = 2(\cos 225^\circ + i \sin 225^\circ) = -1.41 - 1.41i$

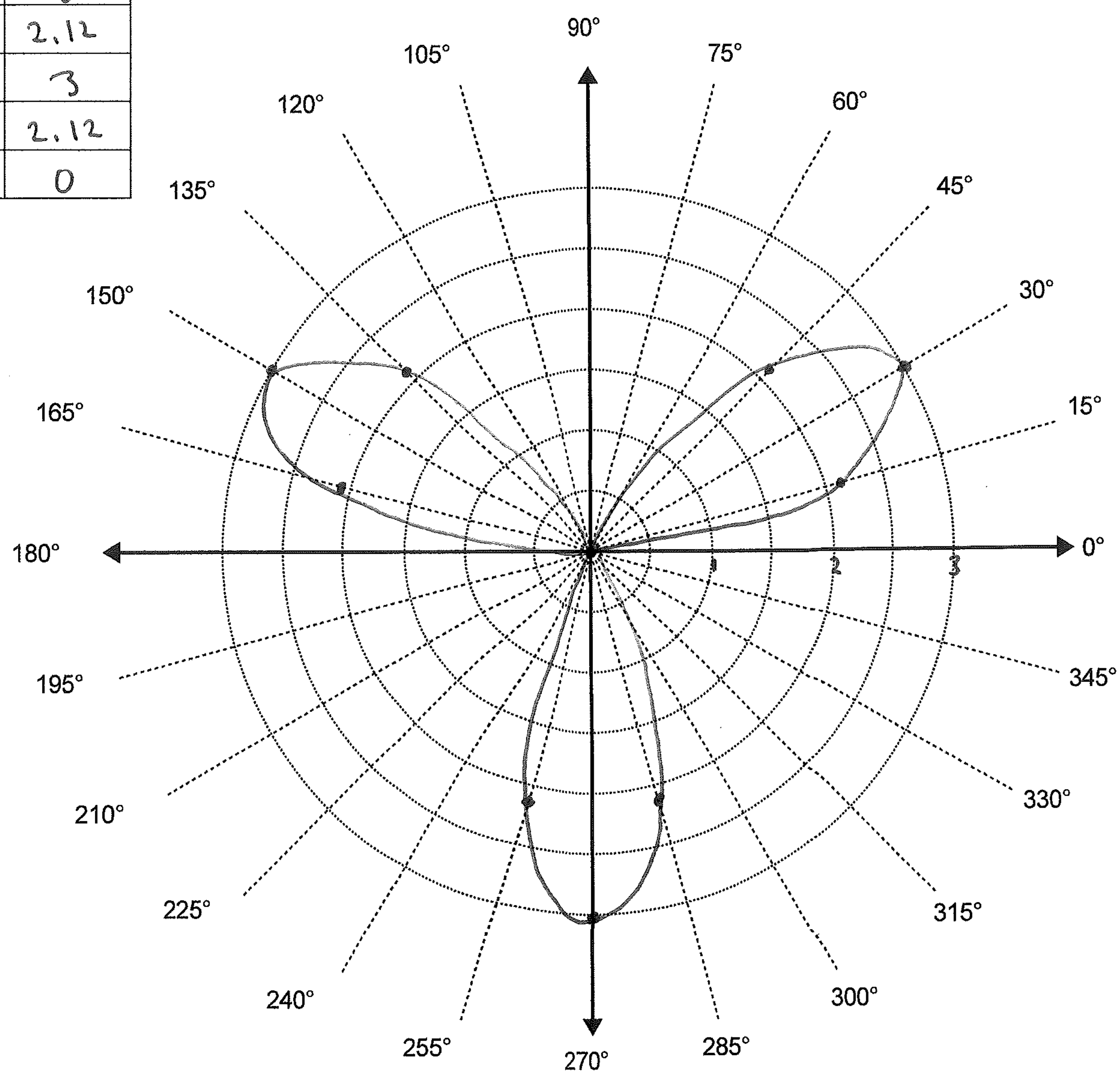
$w_2 = 2(\cos 345^\circ + i \sin 345^\circ) = 1.93 - 0.52i$

6. [15 pts.] Complete the table below by evaluating the equation: $r = 3\sin 3\theta$.

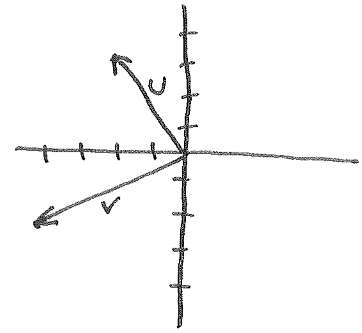
Plot each of the points, and then connect them in order to draw a sketch of the graph.

θ	r
0°	0
15°	2.12
30°	3
45°	2.12
60°	0
75°	-2.12
90°	-3
105°	-2.12
120°	0
135°	2.12
150°	3
165°	2.12
180°	0

Three petal rose



7. [5 pts. each part] Given vectors **u** = $\langle -2,3 \rangle$, and **v** = $\langle -4,-2 \rangle$:



a) find 5**u** - 2**v** = $5\langle -2,3\rangle - 2\langle -4,-2\rangle$

$= \langle -10, 15\rangle + \langle 8,4\rangle$

$= \boxed{\langle -2, 19\rangle}$

b) find the magnitude of **u** (to three decimal places)

$|u| = \sqrt{(-2)^2+(3)^2} = \sqrt{4+9} = \boxed{\sqrt{13}} = 3.606$

c) what is the direction, θ, of **u**?

$\tan\theta = \frac{3}{-2}$

$\tan^{-1}(\tan\theta) = \tan^{-1}(-\frac{3}{2})$

$\bar{\theta} = 56.3^\circ$

$\theta = \boxed{123.7^\circ}$

d) find **u** • **v** =

$u \cdot v = (-2)(-4) + (3)(-2) = 8 + (-6) = \boxed{2}$

* e) calculate the angle between **u** and **v** (to the nearest tenth of a degree)

$|v| = \sqrt{(-4)^2+(-2)^2} = \sqrt{16+4} = \sqrt{20}$

$\cos\theta = \frac{u \cdot v}{|u||v|}$

$\cos\theta = \frac{2}{\sqrt{13}\sqrt{20}} = \frac{2}{\sqrt{260}}$

$\cos^{-1}(\cos\theta) = \cos^{-1}\left(\frac{2}{\sqrt{260}}\right)$

$\boxed{\theta = 82.87^\circ}$