MATH-1150 Exam #4 Instructor: Eric Harshbarger

Student Name: KEY

Show all relevant work (use back of pages for scratch paper, if needed). **CIRCLE FINAL ANSWERS.** Each problem is worth 8 points (and you get one point for clearly writing your full name above).

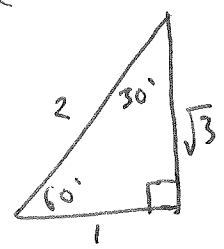
1. Find the exact value of each expression (show answers as fractions not decimals).

a) $\sin(\frac{7\pi}{12})$ $= \sin 105^\circ = \sin(60^\circ+45^\circ) = \sin 60^\circ\cos 45^\circ + \cos 60^\circ \sin 45^\circ$

$\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$\frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \boxed{\frac{\sqrt{6}+\sqrt{2}}{4}}$

$\left(\frac{7\pi}{12}\right)\left(\frac{180^\circ}{\pi}\right) = 105^\circ$



b) $\cot 157.5^\circ = \frac{1}{\tan 157.5^\circ} = \frac{1}{\tan\frac{315^\circ}{2}} = \frac{1}{\frac{\sin 315^\circ}{1+\cos 315^\circ}} = \frac{1+\cos 315^\circ}{\sin 315^\circ} = \frac{1+\frac{\sqrt{2}}{2}}{-\frac{\sqrt{2}}{2}}$

half angle

$= \frac{\frac{2+\sqrt{2}}{2}}{-\frac{\sqrt{2}}{2}} = \frac{2+\sqrt{2}}{-\sqrt{2}} = -\frac{2}{\sqrt{2}} - \frac{\sqrt{2}}{\sqrt{2}} = \boxed{-\sqrt{2}-1}$

$\theta = 45^\circ$

$\cos 315^\circ = \frac{\sqrt{2}}{2}$

$\sin 315^\circ = -\frac{\sqrt{2}}{2}$

2. Write out the Addition Formula for tangent:

$\tan(\theta+\phi) =$ $\frac{\tan\theta + \tan\phi}{1 - \tan\theta\tan\phi}$

3. Write out the Half-Angle Formula for sine:

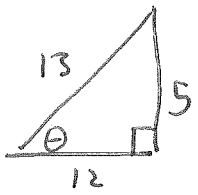
$\sin\frac{\theta}{2} =$ $\pm\sqrt{\frac{1-\cos\theta}{2}}$

4. Circle any of the following expressions (possibly more than one) that is equivalent to $\cot\theta$

$\frac{\cos\theta}{\sin\theta}$ $\tan\left(\frac{\pi}{2}-\theta\right)$ $\csc^2\theta - 1$ $\frac{1}{\tan\theta}$

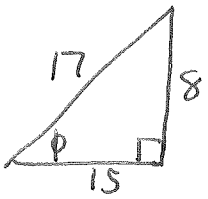
5. Find the exact value (fractions not decimals) of: $\cos\left(\sin^{-1}\frac{5}{13} - \tan^{-1}\frac{8}{15}\right)$.

θ ϕ



$\cos(\theta-\phi) = \cos\theta\cos\phi + \sin\theta\sin\phi$

$\left(\frac{12}{13}\right)\left(\frac{15}{17}\right) + \left(\frac{5}{13}\right)\left(\frac{8}{17}\right)$



$\frac{180}{221} + \frac{40}{221}$

$\boxed{\frac{220}{221}}$

6. Find the exact value (fractions not decimals) of $\cos\frac{x}{2}$ given that $\tan x = \frac{3}{4}$, and $180° < x < 270°$.

x in QIII

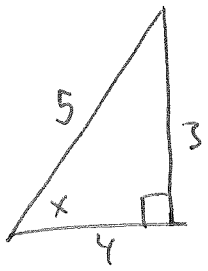
$90° < \frac{x}{2} < 135°$

so $\frac{x}{2}$ in QII

$\cos\frac{x}{2} = -\sqrt{\frac{1+\cos x}{2}} = -\sqrt{\frac{1+\left(\frac{-4}{5}\right)}{2}}$

$= -\sqrt{\frac{1-\frac{4}{5}}{2}} = -\sqrt{\frac{\frac{1}{5}}{\frac{2}{1}}} = -\sqrt{\frac{1}{10}}$

$= \boxed{\frac{1}{\sqrt{10}}}$ or $\frac{-\sqrt{10}}{10}$



7. Circle the one expression below that is equal to: $\dfrac{2}{\tan\theta+\cot\theta}$ = ??

A. $\sin 2\theta$

B. $\sec\theta$

C. 1

D. $\sin^2\theta$

$$\frac{2}{\tan\theta+\cot\theta} = \frac{2}{\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}} = \frac{2}{\frac{\sin^2\theta+\cos^2\theta}{\sin\theta\cos\theta}} = \frac{\frac{2}{1}}{\frac{1}{\sin\theta\cos\theta}} = \left(\frac{2}{1}\right)\left(\frac{\sin\theta\cos\theta}{1}\right) =$$

$$2\sin\theta\cos\theta = \sin 2\theta$$

8. Circle the one expression below that is equal to: $\dfrac{\tan\theta}{\sec\theta}$ = ??

A. $\cot\theta$

B. 0

C. $1+\cos\theta$

D. $\sin\theta$

$$\frac{\tan\theta}{\sec\theta} = \frac{\frac{\sin\theta}{\cos\theta}}{\frac{1}{\cos\theta}} = \left(\frac{\sin\theta}{\cancel{\cos\theta}}\right)\left(\frac{\cancel{\cos\theta}}{1}\right) = \sin\theta$$

9. Circle the one expression below that is equal to: $\cos(x+y)+\cos(x-y)$ = ??

A. $2\cos x\sin y$

B. $\cot(x+y)$

C. $2\cos x$

D. $2\cos x\cos y$

$$\cos(x+y)+\cos(x-y) = \cos x\cos y - \cancel{\sin x\sin y} + \cos x\cos y + \cancel{\sin x\sin y}$$

$$= 2\cos x\cos y$$

10. Find all solutions for θ in the given equations (answers may be in either degrees or radians).

a) $\sin 2\theta + \cos\theta = 0$ (express angles exactly, no decimals)

$2\sin\theta\cos\theta + \cos\theta = 0$

$(\cos\theta)(2\sin\theta + 1) = 0$

$\cos\theta = 0$ or $2\sin\theta + 1 = 0$

$2\sin\theta = -1$

$\sin\theta = -\frac{1}{2}$ $\quad \theta = 30°$

$\theta = 90° + 360°k,\ 270° + 360°k,\ 210° + 360°k,\ 330° + 360°k$

b) $2\sin^2\theta - 1 = 0$ (express angles exactly, no decimals)

$2\sin^2\theta = 1$

$\sin^2\theta = \frac{1}{2}$

$\sin\theta = \pm\sqrt{\frac{1}{2}} = \pm\frac{1}{\sqrt{2}} = \pm\frac{\sqrt{2}}{2}$

$\sin\theta = \frac{\sqrt{2}}{2}$ or $\sin\theta = -\frac{\sqrt{2}}{2}$

$\theta = 45° + 360°k,\ 135° + 360°k,\ 225° + 360°k,\ 315° + 360°k$

or $45° + 90°k$

or solve by:

$2\sin^2\theta - 1 = 0$

$1 - 2\sin^2\theta = 0$

$\cos 2\theta = 0$

$2\theta = 90° + 360°k,\ 270° + 360°k$

$\theta = 45° + 180°k,\ 135° + 180°k$

c) $\cot\frac{\theta}{5} = \frac{1}{\sqrt{3}}$ (express angles exactly, no decimals)

$\tan\frac{\theta}{5} = \sqrt{3}$

$\frac{\theta}{5} = 60° + 180°k$

$\theta = 300° + 900°k$