MATH-1150 Exam #4 Instructor: Eric Harshbarger

Student Name: KEY

Show all relevant work (use back of pages for scratch paper, if needed). **CIRCLE FINAL ANSWERS.** Each problem is worth 8 points (and you get one point for clearly writing your name above).

1. Find the exact value of each expression (show answers as fractions not decimals).

#24 p561

a) $\cos(\frac{3\pi}{8}) = \cos 67.5^\circ = \sqrt{\frac{1+\cos 135^\circ}{2}} = \sqrt{\frac{1+\left(-\frac{\sqrt{2}}{2}\right)}{2}} = \sqrt{\frac{\frac{2-\sqrt{2}}{2}}{\frac{2}{1}}}$

$U = 135^\circ$

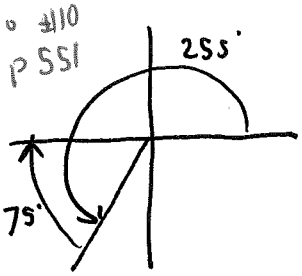
$\frac{U}{2} = 67.5^\circ$

$= \sqrt{\frac{2-\sqrt{2}}{2}\cdot\frac{1}{2}} = \sqrt{\frac{2-\sqrt{2}}{4}} = \sqrt{\frac{1}{4}}\sqrt{2-\sqrt{2}} = \boxed{\frac{1}{2}\sqrt{2-\sqrt{2}}}$

USE HALF-ANGLE FORMULA

similar to #10 p551

b) $\sin 255^\circ = \boxed{-\frac{\sqrt{6}+\sqrt{2}}{4}}$



$\tilde{\theta} = 75^\circ$

$\sin 75^\circ = \sin(45^\circ+30^\circ) = \sin 45^\circ\cos 30^\circ + \cos 45^\circ \sin 30^\circ$

$\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$

$\frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \frac{\sqrt{6}+\sqrt{2}}{4}$

2. Write out a Half-Angle Formula for tangent:

$\tan(\frac{\theta}{2}) = \frac{1-\cos\theta}{\sin\theta}$ or $\frac{\sin\theta}{1+\cos\theta}$

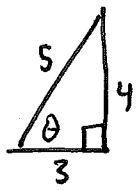
3. Write out the Subtraction Formula for cosine:

$\cos(\theta-\phi) = \cos\theta\cos\phi + \sin\theta\sin\phi$

4. Circle any of the following expressions (possibly more than one) that is equivalent to $\tan^2\theta$

$\frac{\cos^2\theta}{\sin^2\theta}$ $\quad$ $\frac{1}{\cot^2\theta}$ $\quad$ $\sec^2\theta - 1$ $\quad$ $\sec^2\theta + 1$

5. Evaluate $\tan 2\theta$ given that $\sin\theta = \frac{4}{5}$, θ in Quadrant II.

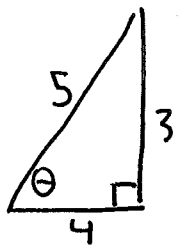


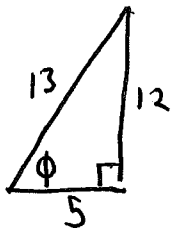
$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta} = \frac{2(-4/3)}{1-(-4/3)^2} = \frac{-8/3}{1-\frac{16}{9}}$$

$$= \frac{-8/3}{-7/9} = \left(-\frac{8}{3}\right)\left(-\frac{9}{7}\right) = \boxed{\frac{24}{7}}$$

6. Find the exact value (fractions not decimals) of: $\sin(\tan^{-1}\frac{3}{4} + \cos^{-1}\frac{5}{13})$

θ $\quad$ ϕ





$$\sin(\theta+\phi) = \sin\theta\cos\phi + \cos\theta\sin\phi$$

$$\left(\frac{3}{5}\right)\left(\frac{5}{13}\right) + \left(\frac{4}{5}\right)\left(\frac{12}{13}\right)$$

#66 p 579

$$\frac{15}{65} + \frac{48}{65}$$

$$\boxed{\frac{63}{65}}$$

7. Find all solutions for θ in the given equations (answers may be in either degrees or radians).

a) $\sin\theta = \cos 2\theta$

$\sin\theta = 2\sin\theta\cos\theta$

$0 = 2\sin\theta\cos\theta - \sin\theta$

$0 = (2\cos\theta - 1)(\sin\theta)$

$\sin\theta = 0$ or $2\cos\theta - 1 = 0$

$\theta = 0° + 360°k$ $\quad$ $\cos\theta = \frac{1}{2}$

$180° + 360°k$ $\quad$ $60° + 360°k$

$300° + 360°k$

$\sin\theta = \cos 2\theta$

$\sin\theta = 1 - 2\sin^2\theta$

$2\sin^2\theta + \sin\theta - 1 = 0$

$(2\sin\theta - 1)(\sin\theta + 1) = 0$

$2\sin\theta - 1 = 0$ or $\sin\theta + 1 = 0$

$\sin\theta = \frac{1}{2}$ or $\sin\theta = -1$

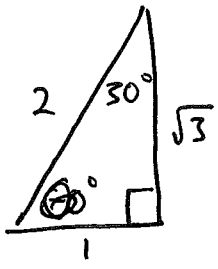
$\theta = 30° + 360°k$ $\quad$ $\theta = 270° + 360°k$

$150° + 360°k$

b) $\tan 5\theta = -\sqrt{3}$

$5\theta = 120° + 180°k$

$\theta = 24° + 36°k$



$\bar{\theta} = 60°$

c) $5\cos\theta + 3 = 7$

$5\cos\theta = 4$

$\cos\theta = 4/5 = 0.8$

$\theta = \cos^{-1}(0.8)$

$\theta = 36.87° + 360°k$

$323.13° + 360°k$

* NOTE: Actual letter choices for answers vary depending on which version of the test you had.

8. Circle the one expression below that is equal to: $\frac{1+\cos\theta}{1+\sec\theta}$ = ??

#21 p543

A. $\cos\theta$

B. $\tan^2\theta$

C. 1

D. $2+\sin\theta$

$$\frac{1+\cos\theta}{1+\sec\theta} = \frac{1+\cos\theta}{1+\frac{1}{\cos\theta}} = \frac{1+\cos\theta}{\frac{\cos\theta}{\cos\theta}+\frac{1}{\cos\theta}} = \frac{1+\cos\theta}{\frac{\cos\theta+1}{\cos\theta}} = \left(\frac{1+\cos\theta}{1}\right)\left(\frac{\cos\theta}{1+\cos\theta}\right) = \boxed{\cos\theta}$$

9. Circle the one expression below that is equal to: $\frac{2}{\cot\theta+\tan\theta}$ = ??

A. $\sin 2\theta$

B. $-\csc\theta$

C. $\cos^2\theta$

D. $2\sin\theta$

$$\frac{2}{\cot\theta+\tan\theta} = \frac{2}{\frac{\cos\theta}{\sin\theta}+\frac{\sin\theta}{\cos\theta}} = \frac{2}{\frac{\cos^2\theta}{\cos\theta\sin\theta}+\frac{\sin^2\theta}{\cos\theta\sin\theta}} = \frac{2}{\frac{\cos^2\theta+\sin^2\theta}{\cos\theta\sin\theta}} = \frac{2}{\frac{1}{\cos\theta\sin\theta}}$$

$$= \left(\frac{2}{1}\right)\left(\frac{\cos\theta\sin\theta}{1}\right) = 2\sin\theta\cos\theta = \boxed{\sin 2\theta}$$

10. Circle the one expression below that is equal to: $\frac{\sin 2\theta}{1-\cos 2\theta}$ = ??

A. $\cot\theta$

B. $1-\cos\theta$

C. $\sec^2\theta$

D. 0

$$\frac{\sin 2\theta}{1-\cos 2\theta} = \frac{2\sin\theta\cos\theta}{1-(1-2\sin^2\theta)} = \frac{2\sin\theta\cos\theta}{1-1+2\sin^2\theta} = \frac{2\sin\theta\cos\theta}{2\sin^2\theta} = \frac{\sin\theta\cos\theta}{\sin\theta\sin\theta}$$

$$= \frac{\cos\theta}{\sin\theta} = \boxed{\cot\theta}$$