MATH-1150 EXAM #3 Instructor: Eric Harshbarger

Student Name: KEY

Show all relevant work (use back of pages for scratch paper, if needed). **CIRCLE FINAL ANSWERS.**

1. [5 pts each] Evaluate the following and express answers in standard form $a + bi$:

a) $\frac{5-i}{-2+3i} \cdot \frac{-2-3i}{-2-3i} = \frac{-10-15i+2i+3i^2}{4+6i-6i-9i^2} = \frac{-13-13i}{13} = \frac{-13}{13} - \frac{13}{13}i$

$= \boxed{-1-i}$

b) $(7-4i) - (-8+3i) = \boxed{15-7i}$

c) $\sqrt{-7}\sqrt{-3} = i\sqrt{7}\,i\sqrt{3} = i^2\sqrt{21} = \boxed{-\sqrt{21}}$

2. [5 pts] Find all solutions of the equation $5x^2-3x+2=0$ and express them in the form $a + bi$.

$a=5$
$b=-3$
$c=2$

$x = \frac{3 \pm \sqrt{(-3)^2-4(5)(2)}}{2(5)} = \frac{3 \pm \sqrt{9-40}}{10} = \frac{3 \pm \sqrt{-31}}{10}$

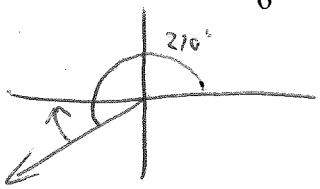
$= \frac{3 \pm i\sqrt{31}}{10} = \boxed{\frac{3}{10} \pm \frac{\sqrt{31}}{10}i}$

3. [30 pts] Complete the table with the exact values (write answers as fractions, *not* decimals) of the trigonometric functions. If a value is undefined, write "undef."

θ in degrees	sin θ	cos θ	tan θ	csc θ	sec θ	cot θ
0°	0	1	0	—	1	—
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$	2	$\frac{2}{\sqrt{3}}$	$\sqrt{3}$
45°	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	$\sqrt{2}$	$\sqrt{2}$	1
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{2}{\sqrt{3}}$	2	$\frac{1}{\sqrt{3}}$
90°	1	0	—	1	—	0

4. [5 pts each] Find the exact (fractions not decimals) value of

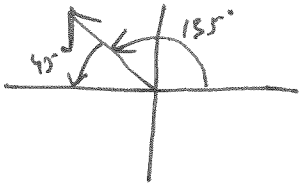
(a) $\cot(-\frac{5\pi}{6}) = \cot(-150°) = \cot(210°) = \frac{1}{\tan 210°} = \frac{1}{\frac{1}{\sqrt{3}}} = \boxed{\sqrt{3}}$



$\theta = 30°$

$\tan 30°$

(b) $\cos(495°) =$ $\cos(135°) = \boxed{-\frac{\sqrt{2}}{2}}$

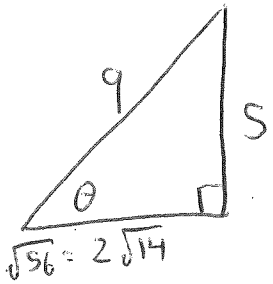


$\theta = 45°$

$\cos 45° = \frac{\sqrt{2}}{2}$

5. [5 pts] Find the exact value (fractions not decimals) of $\cot(\sin^{-1}(\frac{5}{9})) = \boxed{\frac{\sqrt{56}}{5} = \frac{2\sqrt{14}}{5}}$

θ



$9^2 = 5^2 + a^2$

$81 = 25 + a^2$

$a^2 = 56$

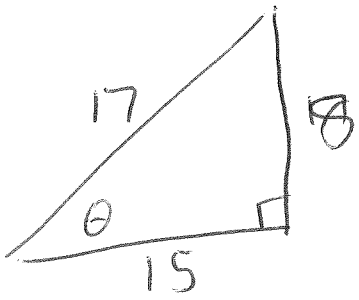
$a = \sqrt{56} = 2\sqrt{14}$

6. [10 pts] Given θ in Quadrant III and the value of $\tan\theta$, find the exact values (fractions not decimals) of the remaining trigonometric functions:

$\sin\theta =$ $-8/17$ $\qquad$ $\csc\theta =$ $-17/8$

$\cos\theta =$ $-15/17$ $\qquad$ $\sec\theta =$ $-17/15$

$\tan\theta =$ $\frac{8}{15}$ $\qquad$ $\cot\theta =$ $\frac{15}{8}$

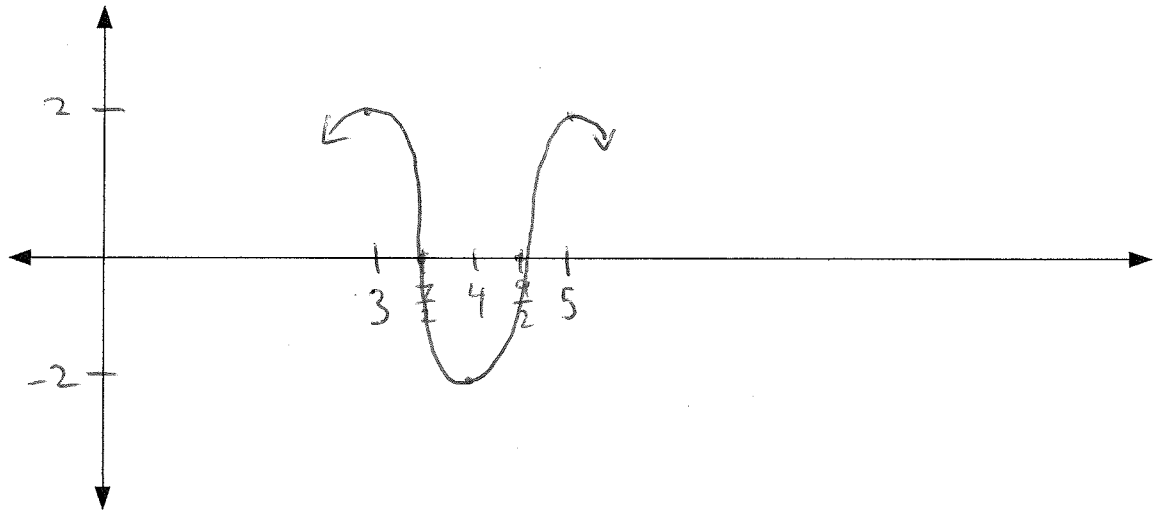


$= 2\cos\pi(x-3)$

7. [10 pts] Find the amplitude, period, and horizontal shift of the function $y = 2\cos(\pi x - 3\pi)$. Then graph one period and label all relevant points along the axes.

Amplitude = $|2| = 2$ $\qquad$ Period = $\frac{2\pi}{\pi} = 2$

Horizontal Shift = 3 to right



8. [10 pts] Given the following information, solve for all possible triangles. If no triangle is possible, write, "Impossible." If two triangles are possible, list both sets of values. Express answers to one decimal place.

ASS

Law of Sines (case 2)

$\angle A = 35.0°$ $\quad a = 12.0$

$\angle B = 22.5°$ ~~157.5°~~ $\quad b = 8.0$

$\angle C = 122.5°$ $\quad c = 17.6$

$\frac{\sin 35°}{12} = \frac{\sin B}{8}$

$\sin B = \frac{8 \sin 35°}{12}$

$B = \sin^{-1}\left(\frac{8 \sin 35°}{12}\right)$

$B = 22.5°$

$\frac{\sin 35°}{12} = \frac{\sin 122.5°}{c}$

$c = \frac{12 \sin 122.5°}{\sin 35°}$

$c = 17.6$

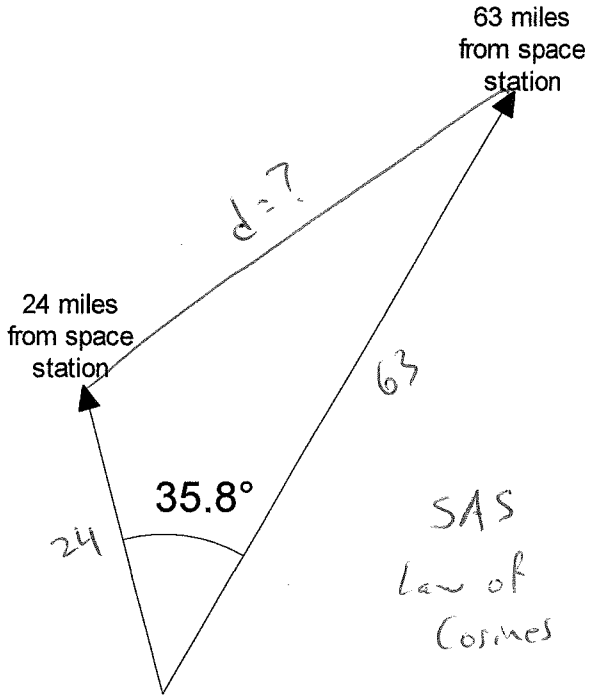
9. [10 pts] Two spaceships depart from a space station such that their trajectories form a 35.8° angle. After a time the first ship has traveled a distance of 24 miles, while the second has gone 63 miles (see diagram). At that moment how far apart are the two spaceships?

$d^2 = 24^2 + 63^2 - 2(24)(63)\cos 35.8°$

$d^2 = 576 + 3969 - 3024\cos 35.8°$

$d^2 = 4545 - 3024\cos 35.8°$

$\boxed{d = 45.7 \text{ miles}}$



SAS

Law of Cosines