MATH-1150 EXAM #2 Instructor: Eric Harshbarger

Student Name: KEY

Show all relevant work (use back of pages for scratch paper, if needed). **CIRCLE FINAL ANSWERS.** Each problem is worth 7 points.

1. Evaluate each expression (do not use a calculator or decimal places):

a) $7\log 100 - \log 1000 = 7(2) - 3 = 14 - 3 = \boxed{11}$

b) $\log_3 5670 - \log_3 7 - \log_3 10 = \log_3 \frac{5670}{7 \cdot 10} = \log_3 \frac{5670}{70}$

$= \log_3 81 = \boxed{4}$

2. Evaluate: $\log_{37} 526599$ (use calculator to four decimal places).

$= \frac{\log 526599}{\log 37} = \boxed{3.6484}$

3. Combine the following into a single logarithm: $3(\log x + 2\log y) - 5\log(z+6)$

$= 3\log x + 6\log y - 5\log(x+6) = \log x^3 + \log y^6 - \log(x+6)^5$

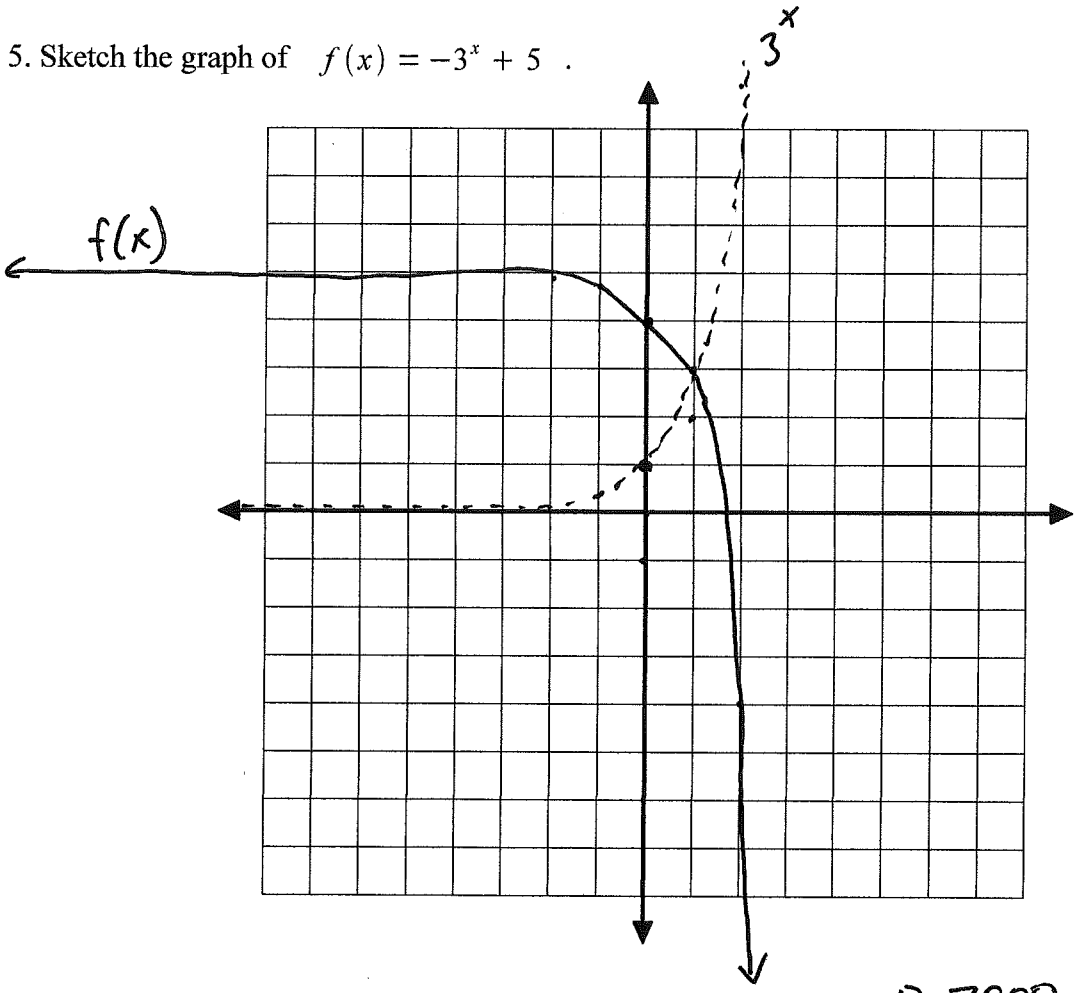
$= \boxed{\log \frac{x^3 y^6}{(x+6)^5}}$

4. Fully expand the following expression using the Laws of Logarithms: $\ln\left(\frac{\sqrt{a}}{b^4 c^7}\right)$

$= \ln\sqrt{a} - \ln b^4 - \ln c^7 = \ln a^{1/2} - \ln b^4 - \ln c^7$

$= \boxed{\frac{1}{2}\ln a - 4\ln b - 7\ln c}$

5. Sketch the graph of $f(x) = -3^x + 5$.



- flip vertically
- up 5

6. A sum of \$7000 is invested at an interest rate of 3% per year.

$P = 7000$

$r = 3\% = 0.03$

a) How much money (to the nearest penny) will be in the account after 18 years if the interest is compounded *monthly*?

$t = 18$

$n = 4$

$$A(t) = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$A(18) = 7000\left(1 + \frac{0.03}{12}\right)^{(12)(18)}$$

$$A(18) = 7000(1.0025)^{216} = \boxed{\$12003.96}$$

b) If the interest is compounded *continuously*, how long will it take for the principal to double? Express your answer in years, to two decimal places.

$t = ?$ $\quad A(t) = 14000$

$$14000 = 7000e^{0.03t}$$

$$2 = e^{0.03t}$$

$$\ln 2 = \ln e^{0.03t}$$

$$\ln 2 = 0.03t$$

$$t = \frac{\ln 2}{0.03} = \boxed{23.10 \text{ years}}$$

7. Find the solution to each equation; you may leave the answer in exact form, or rounded to three decimal places:

a) $13 - \frac{1}{3}\log_4(x+3) = 11$

$-\frac{1}{3}\log_4(x+3) = -2$

$\log_4(x+3) = 6$

$4^6 = x+3$

$4096 = x+3$

$x = 4093$

b) $3^{4x+2} = 1607$

$\log 3^{4x+2} = \log 1607$

$(4x+2)\log 3 = \log 1607$

$4x+2 = \frac{\log 1607}{\log 3}$

$4x = \frac{\log 1607}{\log 3} - 2$

$x = \frac{\frac{\log 1607}{\log 3} - 2}{4} \approx 1.180$

c) $e^{6x} - 10e^{3x} + 21 = 0$

$a = e^{3x}$

$(e^{3x})^2 - 10e^{3x} + 21 = 0$

$a^2 - 10a + 21 = 0$

$(a-3)(a-7) = 0$

$a-3 = 0$ or $a-7 = 0$

$a = 3$ or $a = 7$

$e^{3x} = 3$ or $e^{3x} = 7$

$3x = \ln 3$ $\quad$ $3x = \ln 7$

$x = \frac{\ln 3}{3} \approx 0.366$ $\quad$ $x = \frac{\ln 7}{3} \approx 0.648$

d) $\log_4(2x) = \log_4 3 + \log_4(4-x)$

$\log_4(2x) = \log_4 3(4-x)$

$2x = 3(4-x)$

$2x = 12 - 3x$

$5x = 12$

$x = \frac{12}{5} = 2.4$

$n_0 = 5$

8. A viral outbreak in a small town initially affects 5 people. The relative exponential growth rate of the virus is 12%. $r = 12\% = 0.12$

$n(t) = n_0 e^{rt}$

a) How many people are expected to be infected 30 days later.

$n(30) = 5e^{0.12(30)}$

$= 5e^{3.6} \approx$ **183 people**

$t = ?$ $n(t) = 2000$

b) How long would you expect it to take for 2000 people to become infected (express answer in days, to one decimal place).

$2000 = 5e^{0.12t}$

$400 = e^{0.12t}$

$\ln 400 = 0.12t$

$t = \frac{\ln 400}{0.12} =$ **49.9 days**

9. A wooden artifact discovered at a burial ground contains 16% of the carbon-14 that is present in living trees. How long ago was the artifact made? (The half-life of carbon-14 is 5730 years; express your answer to the nearest year).

$m(t) = m_0 \cdot (1/2)^{t/h}$ $h = 5730$ yrs.

$m(t) = 0.16 m_0$

$0.16 m_0 = m_0 \cdot (1/2)^{t/5730}$

$0.16 = (1/2)^{t/5730}$

$\log(0.16) = \log(0.5)^{t/5730}$

$\log(0.16) = \frac{t}{5730} \log(0.5)$

$t = \frac{5730 \log(0.16)}{\log(0.5)} =$ ~~15339~~ **15149 years**