MATH-1150 EXAM #2 Instructor: Eric Harshbarger

Student Name: KEY

Show all relevant work (use back of pages for scratch paper, if needed). **CIRCLE FINAL ANSWERS.** Each problem is worth 7 points.

1. Evaluate each expression (do not use a calculator or decimal places):

a) $\log 100 + \log 1000$ $= \log_{10} 100 + \log_{10} 1000 = 2 + 3 = \boxed{5}$

b) $\log_5 1750 - \log_5 14 + \log_5 0.2$ $= \log_5 \left(\frac{1750 \cdot 0.2}{14}\right) =$

$\log_5 \left(\frac{350}{14}\right) = \log_5 25 = \boxed{2}$

2. Evaluate: $\log_{93} 726599$ (use calculator to four decimal places).

$= \dfrac{\log 726599}{\log 93} = \boxed{2.9776}$

3. Combine the following into a single logarithm and simplify: $6 \ln(x+1) - 2(\ln y + 5 \ln z)$

$= 6\ln(x+1) - 2\ln y - 10\ln z = \ln(x+1)^6 - \ln y^2 - \ln z^{10}$

$= \boxed{\ln \dfrac{(x+1)^6}{y^2 z^{10}}}$

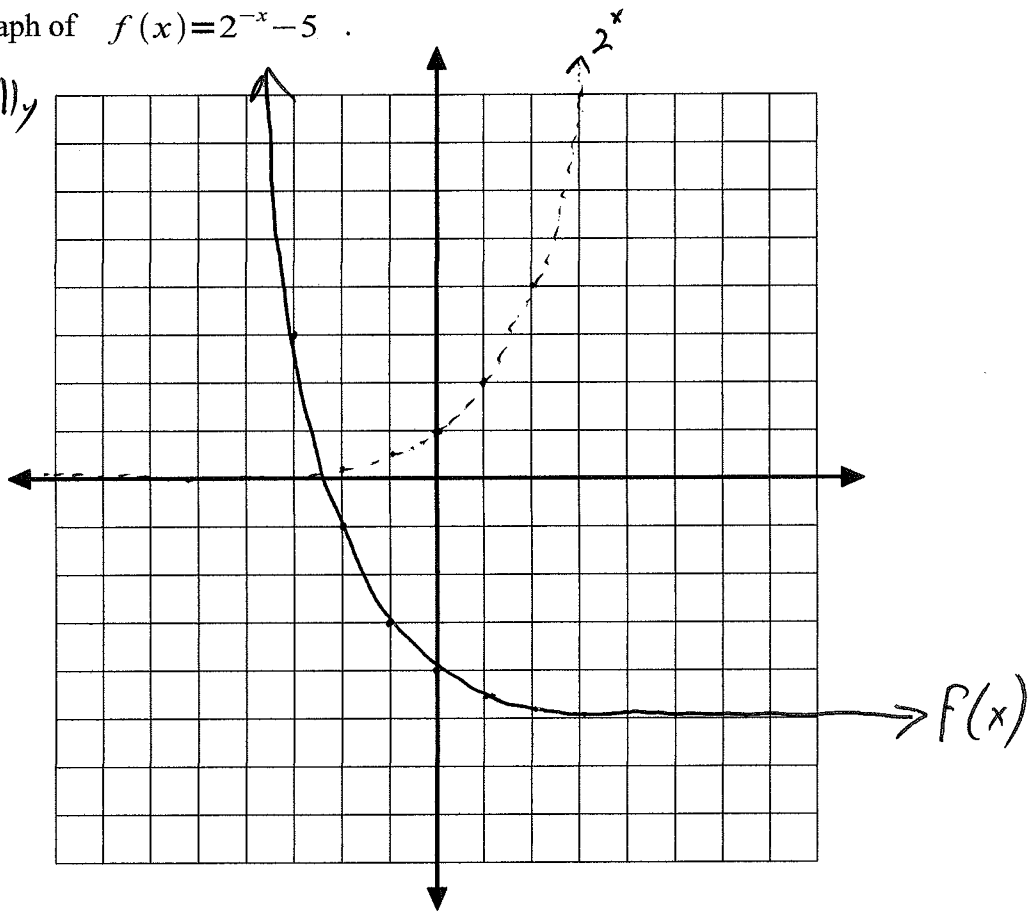
4. Fully expand the following expression using the Laws of Logarithms: $\log\left(\dfrac{ac^2}{b^3 d^7}\right)$

$= \log a + \log c^2 - \log b^3 - \log d^7$

$= \boxed{\log a + 2\log c - 3\log b - 7\log d}$

5. Sketch the graph of $f(x) = 2^{-x} - 5$.

1. flip horizontally
2. down 5



6. A sum of \$2500 is invested at an interest rate of 6% per year.

$P = 2500$
$r = 6\% = 0.06$

$n = 12$ a) How much money (to the nearest penny) will be in the account after 17 years if the interest is compounded *monthly*?

$$A(17) = 2500\left(1 + \frac{0.06}{12}\right)^{12(17)} = 2500(1.005)^{204}$$

$$= \boxed{\$6915.39}$$

b) If the interest is compounded *continuously*, how long will it take for the amount of money to triple? Express your answer in years, to one decimal place.

$$7500 = 2500e^{0.06t}$$
$$3 = e^{0.06t}$$
$$\ln 3 = \ln e^{0.06t}$$
$$\ln 3 = 0.06t$$

$$\rightarrow t = \frac{\ln 3}{0.06} = \boxed{18.3 \text{ years}}$$

7. Find the solution to each equation; you may leave the answer in exact form, or rounded to three decimal places:

a) $\log_4(2x) = \log_4 3 + \log_4(4-x)$

$\log_4(2x) = \log_4 3(4-x)$

$2x = 3(4-x)$

$2x = 12 - 3x$

$5x = 12$

$x = \frac{12}{5} = \boxed{2.4}$

b) $7^{4x+2} = 16082$

$\log 7^{4x+2} = \log 16082$

$(4x+2)\log 7 = \log 16082$

$4x+2 = \frac{\log 16082}{\log 7}$

$4x = \frac{\log 16082}{\log 7} - 2$

$x = \frac{\frac{\log 16082}{\log 7} - 2}{4} \approx \boxed{0.744}$

c) $e^{4x} - 7e^{2x} + 12 = 0$

let $a = e^{2x}$

$(e^{2x})^2 - 7e^{2x} + 12 = 0$

$a^2 - 7a + 12 = 0$

$(a-3)(a-4) = 0$

$a - 3 = 0$ or $a - 4 = 0$

$a = 3$; $a = 4$

$e^{2x} = 3$; $e^{2x} = 4$

$2x = \ln 3$; $2x = \ln 4$

$\boxed{x = \frac{\ln 3}{2} \approx 0.549 \text{ or } x = \frac{\ln 4}{2} \approx 0.693}$

d) $5 - \frac{1}{2}\log_5(x+3) = 2$

$-\frac{1}{2}\log_5(x+3) = -3$

$\log_5(x+3) = 6$

$5^6 = x+3$

$15625 = x+3$

$\boxed{x = 15622}$

8. A viral outbreak in a small town initially affects 14 people. The relative exponential growth rate of the virus is 3% per day.

$n_0 = 14$

$r = 0.03$

a) How many people are expected to be infected 30 days later.

$n(t) = n_0 e^{rt}$

$n(t) = 14e^{0.03t}$

$n(30) = 14e^{0.03(30)} = 14e^{0.9} = \boxed{34 \text{ people}}$

b) How long would you expect it to take for 2000 people to become infected (express answer in days, to one decimal place).

$2000 = 14e^{0.03t}$

$\frac{2000}{14} = e^{0.03t}$

$\ln\left(\frac{1000}{7}\right) = \ln e^{0.03t}$

$\ln\left(\frac{1000}{7}\right) = 0.03t$

$t = \frac{\ln\left(\frac{1000}{7}\right)}{0.03} \approx \boxed{165.4 \text{ days}}$

9. A wooden artifact discovered at a burial ground contains 77% of the carbon-14 that is present in living trees. How long ago was the artifact made? (The half-life of carbon-14 is 5730 years; express your answer in years, to one decimal place).

$m(t) = 0.77m_0$

$h = 5730$

$m(t) = m_0 \cdot 2^{-t/5730}$

$0.77\cancel{m_0} = \cancel{m_0} \cdot 2^{-t/5730}$

$0.77 = 2^{-t/5730}$

$\log(0.77) = \log 2^{-t/5730}$

$\log(0.77) = \frac{-t}{5730}\log 2$

$\frac{-5730\log(0.77)}{\log 2} = t \approx \boxed{2160.6 \text{ years}}$