MATH-1150 EXAM #2 Instructor: Eric Harshbarger

Student Name: KEY

Show all relevant work (use back of pages for scratch paper, if needed). **CIRCLE FINAL ANSWERS.** All answers are worth 7 points each.

1. Evaluate each expression (write answers exact, no decimal places):

a) $\log_{25} 60 - \log_{25} 3 - \log_{25} 4 = \log_{25}\left(\frac{60}{3\cdot 4}\right) = \log_{25}\left(\frac{60}{12}\right)$

$= \log_{25} 5 = \boxed{1/2}$

b) $\log_3 27 - 2\log_3 9 = \log_3 27 - \log_3 9^2 = \log_3 27 - \log_3 81$

$= \log_3 \frac{27}{81} = \log_3 \frac{1}{3} = \boxed{-1}$

2. Use a calculator to evaluate $\log_{79} 1600$ rounded to four decimal places:

$= \frac{\ln 1600}{\ln 79} = \boxed{1.6885}$

3. Combine $3\log(x-4)+\log(x+2)-\log(x^2-x-12)$ into a single logarithm and simplify, if possible:

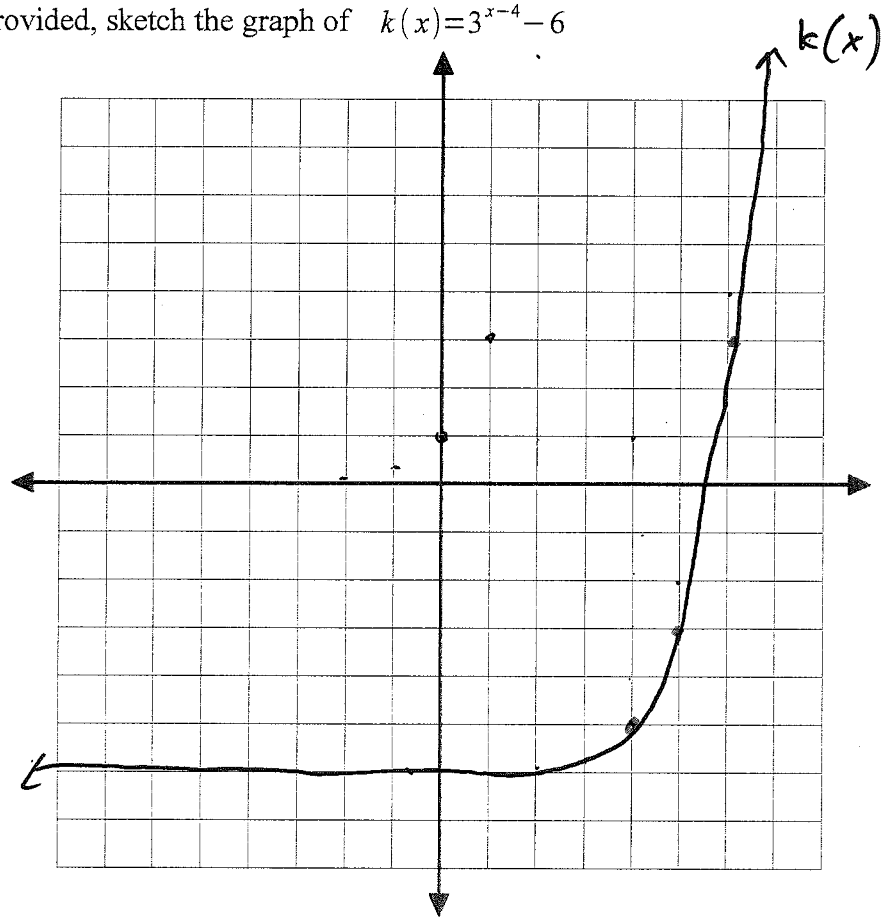
$= \log(x-4)^3 + \log(x+2) - \log(x^2-x-12)$

$= \log \frac{(x-4)^3(x+2)}{x^2-x-12} = \log \frac{(x-4)^3(x+2)}{(x-4)(x+3)} = \boxed{\log \frac{(x-4)^2(x+2)}{(x+3)}}$

4. In a small pond the population of fish follows the following growth model: $P(t)=\frac{1200}{1+11e^{-0.2t}}$. Where $t$ is measured in years. How many fish are expected to be in the pond after 15 years?

$P(15) = \frac{1200}{1+11e^{-0.2(15)}} = \boxed{775 \text{ fish}}$

5. On the grid provided, sketch the graph of $k(x)=3^{x-4}-6$




1. right 4
2. down 6

6. A sum of \$2500 is invested at an interest rate of 6% per year and is compounded *semi-annually* (that is, twice each year).

$P = 2500$
$r = 0.06$
$n = 2$

a) How much money (to the nearest penny) will be in the account after 10 years?

$$A(10) = 2500\left(1+\frac{0.06}{2}\right)^{2(10)}$$
$$= 2500(1.03)^{2(10)}$$
$$= 2500(1.03)^{20}$$
$$\boxed{= \$4515.28}$$

b) How long (in years, to two decimal places) will it take for the account to reach \$10,000?

$$10000 = 2500\left(1+\frac{0.06}{2}\right)^{2t}$$
$$4 = (1.03)^{2t}$$
$$\log 4 = 2t\log(1.03)$$
$$t = \frac{\log 4}{2\log(1.03)} \approx \boxed{23.45 \text{ years}}$$

7. The initial population of stray cats in a village is 35 and doubles every 17 months.

$n_0 = 35$

a) How many cats would you expect there to be after 5 years?

5 years = 60 months $a = 17$ months

$n(t) = 35 \cdot 2^{t/17}$

$n(60) = 35 \cdot 2^{60/17} = \boxed{404 \text{ cats}}$

b) Calculate the relative growth rate, *r*, of the cat population.

$n(t) = n_0 e^{rt}$

$70 = 35 e^{r17}$

$2 = e^{17r}$

$\ln 2 = 17r$

$r = \frac{\ln 2}{17} = \boxed{0.0408 = 4.08\%}$

or

49% if in the context of years

8. In a chemistry laboratory a 200 g sample of Iodine-135 is studied. 28 days later, the sample has decayed down to 17.68 g. From this observation calculate the half-life of Iodine-135.

$m(t) = m_0 \cdot \left(\frac{1}{2}\right)^{t/h}$

$17.68 = 200 \cdot (0.5)^{28/h}$

$\frac{17.68}{200} = (0.5)^{28/h}$

$\log\left(\frac{17.68}{200}\right) = \frac{28}{h} \log(0.5)$

$\frac{\log\left(\frac{17.68}{200}\right)}{\log(0.5)} = \frac{28}{h}$

$28 \log(0.5) = h \log\left(\frac{17.68}{200}\right)$

$h = \frac{28 \log(0.5)}{\log\left(\frac{17.68}{200}\right)} = \boxed{8 \text{ days}}$

9. Find the solution to each equation; you may leave the answer in exact form, or rounded to three decimal places:

a) $e^{6x}-8e^{3x}+15=0$

$a=e^{3x}$ $\quad (e^{3x})^2-8e^{3x}+15=0$

$a^2-8a+15=0$

$(a-3)(a-5)=0$

$a=3 \quad a=5$

$e^{3x}=3$ or $e^{3x}=5$

$3x=\ln 3 \qquad 3x=\ln 5$

$\boxed{x=\frac{\ln 3}{3} \text{ or } x=\frac{\ln 5}{3} \qquad 0.366 \text{ or } 0.536}$

b) $\log_4(x+7)-\log_4(x-2)=3$

$\log_4\left(\frac{x+7}{x-2}\right)=3$

$4^3=\frac{x+7}{x-2}$

$64=\frac{x+7}{x-2}$

$64(x-2)=x+7$

$64x-128=x+7$

$63x=135$

$\boxed{x=\frac{135}{63}=\frac{15}{7}\approx 2.143}$

c) $3+4e^{5x-46}=69751$

$4e^{5x-46}=69748$

$e^{5x-46}=17437$

$5x-46=\ln 17437$

$5x=\ln 17437+46$

$\boxed{x=\frac{\ln 17437+46}{5}\approx 11.153}$

d) $3^{2x+3}=9^{5-x}$

$3^{2x+3}=(3^2)^{5-x}$

$3^{2x+3}=3^{2(5-x)}$

$3^{2x+3}=3^{10-2x}$

one-to-one so $\longrightarrow$ $2x+3=10-2x$

$4x=7$

$\boxed{x=7/4=1.75}$