

Student Name: KEY

Show all relevant work (use back of pages for scratch paper, if needed). **CIRCLE FINAL ANSWERS.**
 Leave answers exact (not decimals from a calculator).

1. Let $f(x) = \frac{\sqrt{8-2x}}{x-3}$.

"Any real number less than or equal to 4 except 3."

(a) [6 pts.] What is the domain of f ?
 problems when:

$$\begin{aligned} x-3=0 & \quad 8-2x < 0 \\ x=3 & \quad -2x < -8 \\ & \quad x > 4 \end{aligned}$$

$$\{x \in \mathbb{R} \mid x \leq 4, x \neq 3\}$$

$$(-\infty, 3) \cup (3, 4]$$

(b) [6 pts.] Evaluate $f(-4)$.

$$f(-4) = \frac{\sqrt{8-(2)(-4)}}{-4-3} = \frac{\sqrt{8+8}}{-7} = \frac{\sqrt{16}}{-7} = \boxed{\frac{-4}{7}}$$

2. [7 pts.] Given function $C(x) = 5 - 2x^2$, what is the average rate of change of C over the interval between $x = -3$ and $x = 2$?

$$\begin{aligned} \frac{C(b)-C(a)}{b-a} &= \frac{(5-2(2)^2)-(5-2(-3)^2)}{2-(-3)} = \frac{(5-2(4))-(5-2(9))}{5} = \\ &= \frac{(5-8)-(5-18)}{5} = \frac{-3-(-13)}{5} = \frac{-3+13}{5} = \frac{10}{5} = \boxed{2} \end{aligned}$$

3. Let $f(x) = \begin{cases} 7 + 3x^3 & \text{if } x \leq 2 \\ 12 - x & \text{if } 2 < x \leq 5 \\ x^2 - 2 & \text{if } x > 5 \end{cases}$

(a) [6 pts.] Evaluate $f(2)$.

$$f(2) = 7 + 3(2)^3 = 7 + 3(8) = 7 + 24 = \boxed{31}$$

(b) [6 pts.] Evaluate $f(6)$.

$$f(6) = 6^2 - 2 = 36 - 2 = \boxed{34}$$

4. The rule for function g is described in words as: "Add 3 to the input number, take the absolute value, then multiply by 2."

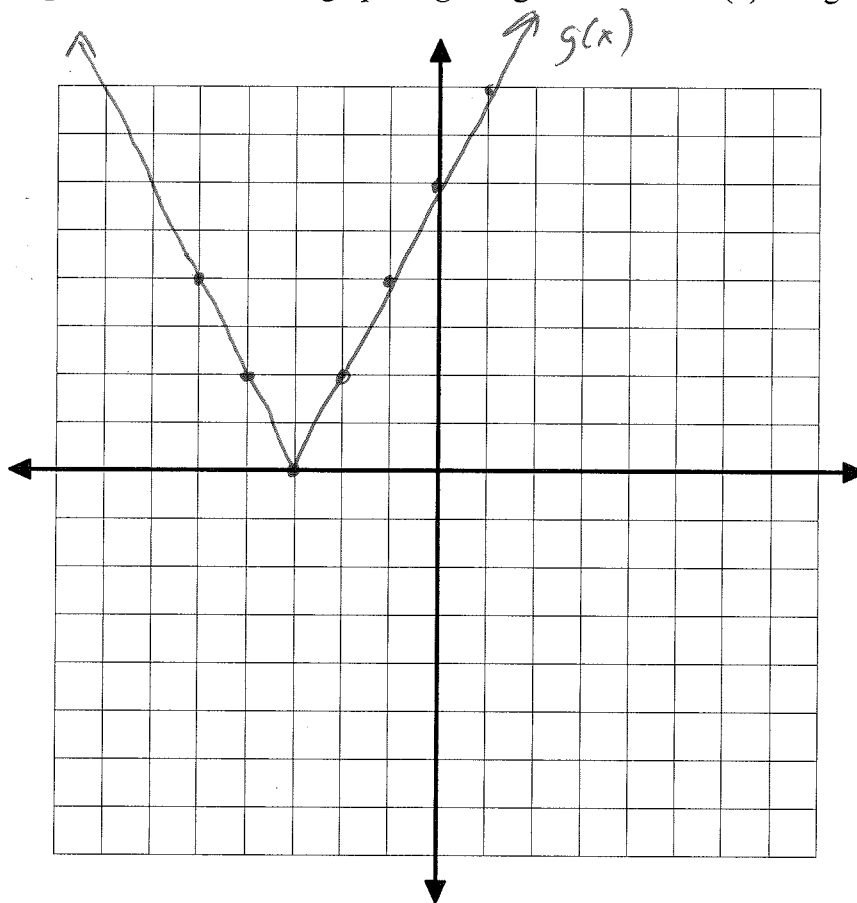
(a) [6 pts.] Express function g algebraically:

$$g(x) = 2|x+3|$$

(b) [7 pts.] Complete the chart of values for g :

x	$g(x)$
-5	4
-4	2
-3	0
-2	2
-1	4
0	6
1	8

(c) [6 pts.] Plot the points and sketch the graph of g using the table from (b) as a guide.



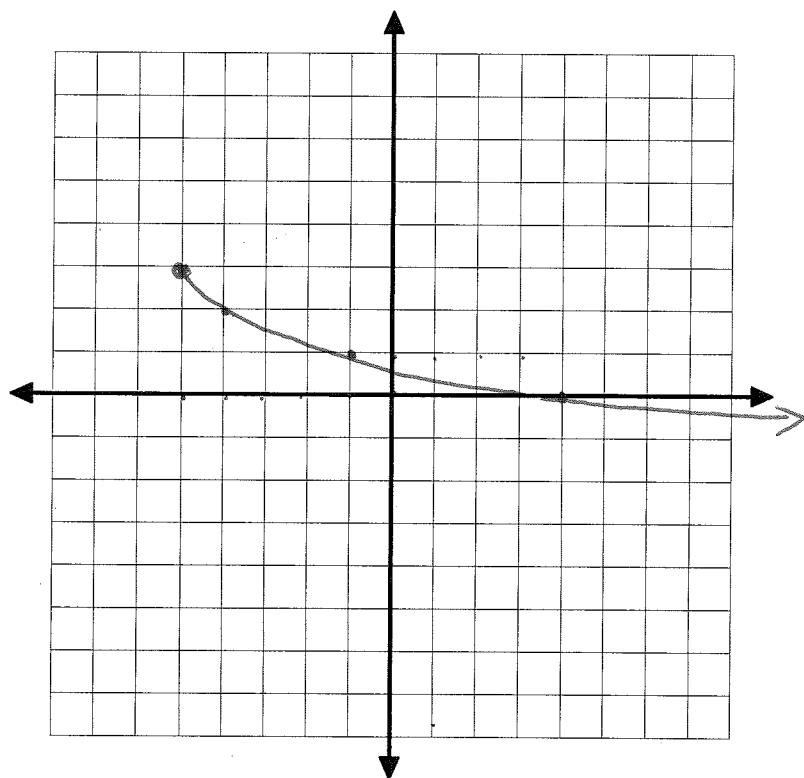
5. [6 pts., each part] If $f(x) = 3x^2 + 4$ and $g(x) = 7 - 2x$, find the following, and simplify:

(a) $(fg)(-1) = f(-1)g(-1) = (3(-1)^2 + 4)(7 - 2(-1)) = (3(1) + 4)(7 + 2)$
 $= (3 + 4)(9) = (7)(9) = \boxed{63}$

(b) $(f-g)(a) = f(a) - g(a) = (3a^2 + 4) - (7 - 2a) = 3a^2 + 4 - 7 + 2a$
 $= \boxed{3a^2 + 2a - 3}$

(c) $(g \circ g)(3) = g(g(3)) = g(7 - 2(3)) = g(7 - 6) = g(1) = 7 - 2(1) =$
 $7 - 2 = \boxed{5}$

6. [7 pts.] Sketch and label the graph of $f(x) = -\sqrt{x+5} + 3$. You may plot points by hand or use knowledge of transformations.



- ☞ square root function
- left 5
 - flip vertically
 - up 3

7. [6 pts., each part] In each part find f^{-1} .

(a) $f(x) = \sqrt[3]{5x+2}$

$$y = \sqrt[3]{5x+2}$$

$$x = \sqrt[3]{5y+2}$$

$$x^3 = 5y+2$$

$$x^3 - 2 = 5y$$

$$y = \frac{x^3 - 2}{5}$$

$$f^{-1}(x) = \frac{x^3 - 2}{5}$$

(b) $f(x) = \frac{2x-1}{3x+7}$

$$y = \frac{2x-1}{3x+7}$$

$$x = \frac{2y-1}{3y+7}$$

$$2y-1 = x(3y+7)$$

$$2y-1 = 3xy+7x$$

$$2y - 3xy = 7x + 1$$

$$y(2-3x) = 7x+1$$

$$y = \frac{7x+1}{2-3x}$$

$$f^{-1}(x) = \frac{7x+1}{2-3x}$$

8. [6 pts., each part] Charlie has a job cleaning widgets at Doodad, Inc. Every morning it takes him 20 minutes to prepare his cleaning station. After that he is able to clean one widget every 8 minutes.

(a) Construct a function, T , that calculates how many minutes it will take Charlie to clean n number of widgets starting from the beginning of his workday.

$$T(n) = 20 + 8n$$

(b) Now, determine $T^{-1}(n)$; that is, the inverse of the function from part (a).

$$y = 20 + 8n$$

$$n = 20 + 8y$$

$$n - 20 = 8y$$

$$y = \frac{n-20}{8}$$

$$T^{-1}(n) = \frac{n-20}{8}$$

(c) Calculate $T^{-1}(420)$. What does this represent?

$$T^{-1}(420) = \frac{420-20}{8} = \frac{400}{8} = 50.$$

During the first 420 minutes (7 hours) of his workday, Charlie can clean 50 widgets.