

Student Name: KEY

Show all relevant work (use back of pages for scratch paper, if needed). **CIRCLE FINAL ANSWERS.** Leave answers exact (not decimals from a calculator). Each problem is worth 5 points.

$$1. \text{ Let } f(x) = \frac{\sqrt{x^2+4}}{x^2-x-6} = \frac{\sqrt{x^2+4}}{(x-3)(x+2)}$$

(a) What is the domain of  $f$ ?

$$\{x \mid x \neq -2, 3\} \text{ or } (-\infty, -2) \cup (-2, 3) \cup (3, +\infty)$$

(b) Evaluate  $f(2)$ .

$$f(2) = \frac{\sqrt{2^2+4}}{2^2-2-6} = \frac{\sqrt{4+4}}{4-8} = \frac{\sqrt{8}}{-4} = \frac{-2\sqrt{2}}{4} = \boxed{\frac{-\sqrt{2}}{2}}$$

2. Given function  $d(t) = 2t^2 - 3$  between  $t = -1$  and  $t = 4$ ,

(a) what is the net change of  $d$ ?

$$\begin{aligned} d(b) - d(a) &= (2(4)^2 - 3) - (2(-1)^2 - 3) = (2(16) - 3) - (2(1) - 3) \\ &= (32 - 3) - (2 - 3) = 29 - (-1) = \boxed{30} \end{aligned}$$

(b) what is the average rate of change of  $d$ ?

$$\frac{d(b) - d(a)}{b - a} = \frac{30}{4 - (-1)} = \frac{30}{5} = \boxed{6}$$

$$3. \text{ Let } h(x) = \begin{cases} 1 + 2x^2 & \text{if } x \leq -3 \\ 7 - x & \text{if } x > -3 \end{cases}$$

(a) Evaluate  $h(5)$ .

$$h(5) = 7 - 5 = \boxed{2}$$

(b) Evaluate  $h(-3)$ .

$$h(-3) = 1 + 2(-3)^2 = 1 + 2(9) = 1 + 18 = \boxed{19}$$

4. A function  $g$  is described in words as: "Subtract 2 from the input, take the absolute value, then subtract 5."

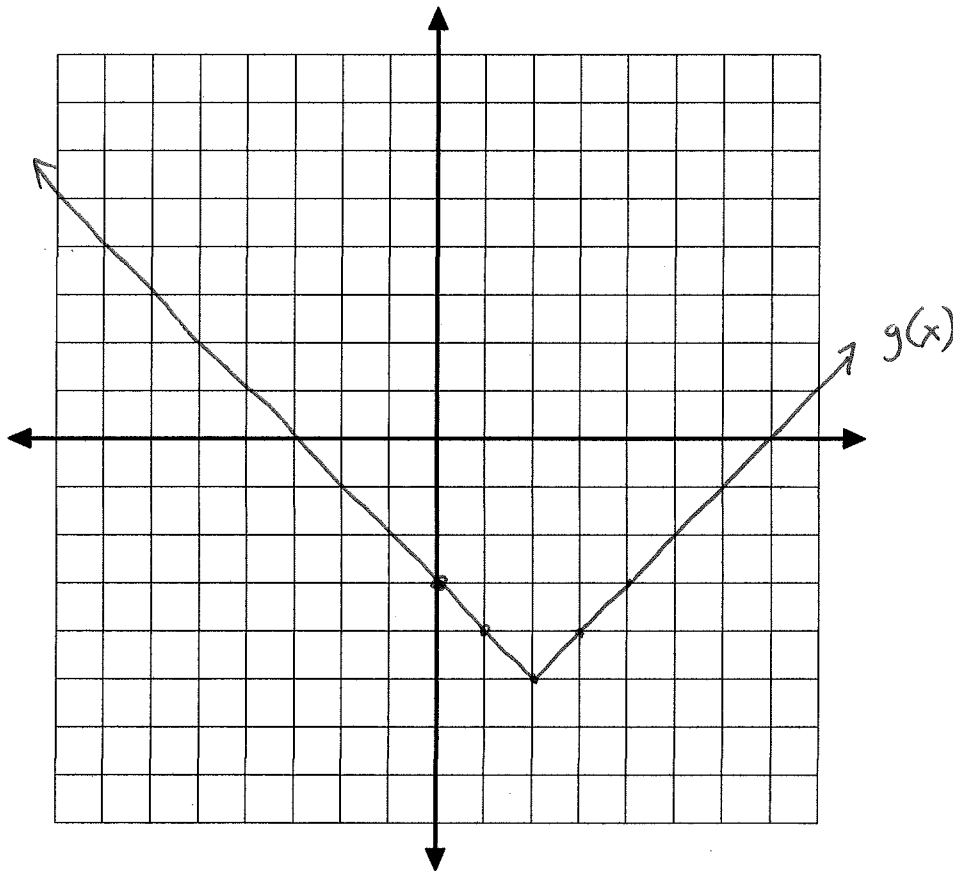
(a) Express function  $g$  algebraically.

$$g(x) = |x - 2| - 5$$

(b) Complete the chart of values for  $g$ :

$x$	$g(x)$
0	-3
1	-4
2	-5
3	-4
4	-3

(c) Plot the points and sketch the graph of  $g$  using the table from (b) as a guide.



5. If  $f(x) = 2 - 3x$  and  $g(x) = x^2 - 5$ , find the following, and simplify :

$$(a) \quad (f-g)(8) = f(8) - g(8) = (2 - 3(8)) - (8^2 - 5) = (2 - 24) - (64 - 5) = (-22) - (59) = \boxed{-81}$$

$$(b) \quad (g+f)(a) = g(a) + f(a) = (a^2 - 5) + (2 - 3a) = \boxed{a^2 - 3a - 3}$$

$$(c) \quad (f \circ f)(-3) = f(f(-3)) = f(2 - 3(-3)) = f(2 + 9) = f(11) = 2 - 3(11) = 2 - 33 = \boxed{-31}$$

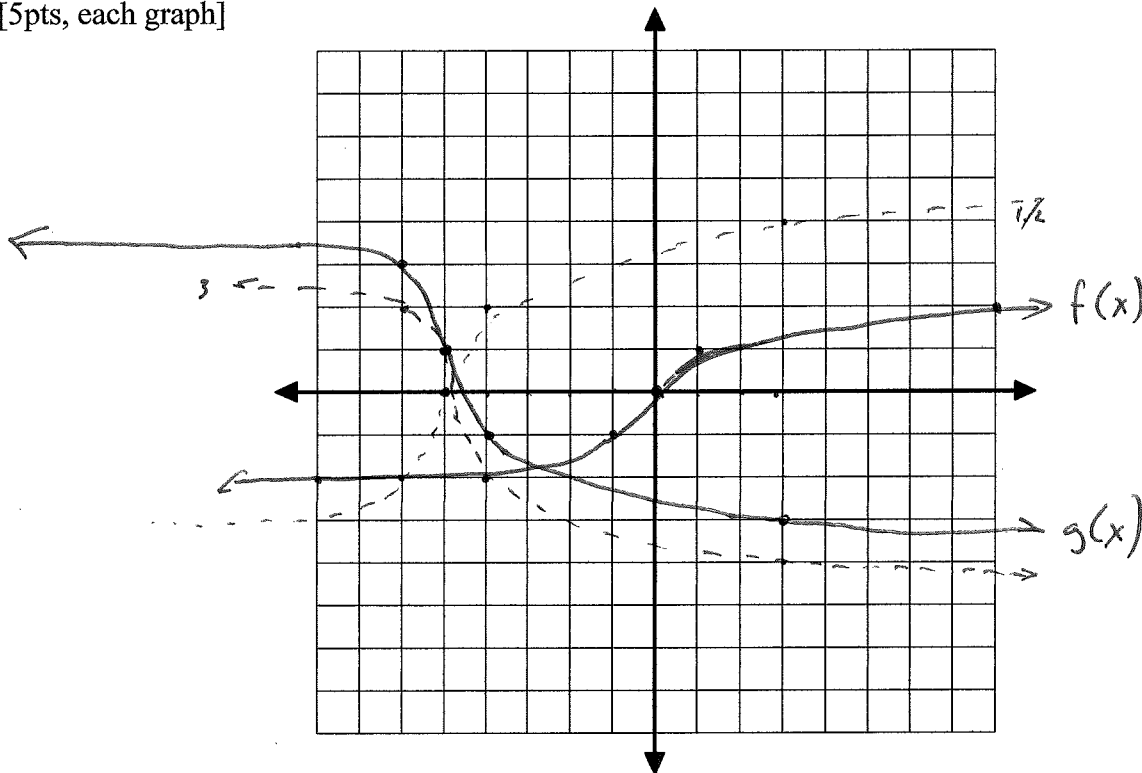
$$(d) \quad f(g(7)) = f(7^2 - 5) = f(49 - 5) = f(44) = 2 - 3(44) = \boxed{-130}$$

6. On the grid below sketch the graph of  $f(x) = \sqrt[3]{x}$ .

Then, sketch the graph of  $g(x) = 1 - 2\sqrt[3]{x+5}$ . =  $-2\sqrt[3]{x+5} + 1$

You may plot points by hand or use knowledge of transformations. Please label each graph drawn.

[5pts, each graph]



- g
1. left 5
  2. stretch x2 vertically
  3. flip up/down
  4. up 1

7. In each part find  $f^{-1}$ .

(a)  $f(x) = \sqrt[7]{1-6x}$

$$y = \sqrt[7]{1-6x}$$

$$x = \sqrt[7]{1-6y}$$

$$x^7 = 1-6y$$

$$x^7 - 1 = -6y$$

$$y = \frac{1-x^7}{6} = f^{-1}(x)$$

(b)  $f(x) = \frac{5x-7}{4x+3}$

$$y = \frac{5x-7}{4x+3}$$

$$x = \frac{5y-7}{4y+3}$$

$$5y-7 = x(4y+3)$$

$$5y-7 = 4xy+3x$$

$$5y-4xy = 3x+7$$

$$y(5-4x) = 3x+7$$

$$y = \frac{3x+7}{5-4x} = f^{-1}(x)$$

8. A taxicab ride will cost you six dollars plus an additional twenty-five cents per minute driven.

(a) Construct a function,  $f$ , that models the cost of a taxicab ride that travels  $x$  number of minutes.

$$f(x) = 6 + 0.25x$$

(b) Calculate how much a 20 minute taxicab ride will cost you.

$$f(20) = 6 + 0.25(20) = 6 + 5 = \boxed{\$11.00}$$

(c) Determine  $f^{-1}(x)$ ; that is, the inverse of the function from part (a).

$$y = 6 + 0.25x$$

$$x = 6 + 0.25y$$

$$x-6 = 0.25y$$

$$y = 4(x-6)$$

$$f^{-1}(x) = 4(x-6)$$

(d) Calculate  $f^{-1}(36)$ . What does this represent?

$$f^{-1}(36) = 4(36-6) = 4(30) = 120$$

For \$36.00 you may take a 120 minute taxicab ride.