Immersions of L_p spaces into Lipschitz subspaces of continuous functions and duality theorems.

Miguel Antonio Jiménez-Pozo
Autonomous University of Puebla , Mexico
&
Geraldo Soares de Souza
Auburn University, USA

Abstract: Let $(\Omega, \mathbb{A}, \mu)$ be a positive finite measure space and X the metric subset of $L^1(\mu)$ of all characteristic functions of measurable sets 1_A. By identifying any pair of measurable sets A, B, whenever $\mu(A\Delta B) = 0$, one has

$$\forall 1_A, 1_B \in \mathbb{X}, \int |1_A - 1_B| d\mu = \mu(A\Delta B) = d(A, B)$$

The real Banach space $L^1(\mu)$ is embedded into a subspace C_μ of $C(X)$ determined by the operator

$$\forall g \in L^1(\mu) \forall A \in \mathbb{A}, \Lambda(g, A) = \int_A g d\mu$$

Since $\|\Lambda\| = 1$ and $\|\Lambda^{-1}\| \leq 2$, $L^1(\mu)$ and C_μ are isomorphic and C_μ is closed in $C(X)$. In this talk we study spaces with Hölder norm with functions in C_μ that satisfy some Lipschitz conditions as well as duality theorems. This is related to Lp, Lorentz and De Souza spaces and others.