Problems

1. Do problem 2 on p. 532 of your textbook, repeat also for the backward Euler method.

2. Use Euler’s method to solve:
   
   (a) \( y' = y, \quad y(0) = 1 \).

   (b) \( y' = 1 + (y - t)^2, \quad y(0) = 1 \).

   Find \( y(0.2) \) using \( h = 0.1 \). Compare to the exact solutions which are \( y = e^t \) and \( y = t + \frac{1}{1-t} \), respectively.

Program

1. Program Euler’s method.

2. Program Heun’s method.

3. Program fourth order Runge-Kutta method.

Apply all methods, using \( h = 0.1 \) to the equations:

- \( y'(t) = y(t), \quad 0 \leq t \leq 3, \quad y(0) = 1 \), which has the exact solution \( y = e^t \).
- \( y'(t) = y(t) - 1, \quad 0 \leq t \leq 3, \quad y(0) = 0 \), which has the exact solution \( y = 1 - e^t \).

Format the output in a table with following headers:

<table>
<thead>
<tr>
<th>( t_i )</th>
<th>Euler Error</th>
<th>Heun Error</th>
<th>RK-4 Error</th>
</tr>
</thead>
</table>

For the errors use the absolute value of the difference between the approximate solution and the exact solution.

Extra. Try to repeat the above using \( h = 0.1 \) for the fourth order Runge-Kutta method, \( h = 0.05 \) for Heun’s method, and \( h = 0.025 \) for Euler’s method.