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**Math-5630/6630**  
Introduction to Numerical Analysis I  
Summer 2007  
Homework 10

**Problems**

1. Do problem 2 on p. 532 of your textbook, repeat also for the backward Euler method.

2. Use Euler's method to solve:

(a) 
$$y' = y, \quad y(0) = 1.$$

(b) 
$$y' = 1 + (y - t)^2, \quad y(0) = 1.$$

Find  $y(0.2)$  using  $h = 0.1$ . Compare to the exact solutions which are  $y = e^t$  and  $y = t + \frac{1}{1-t}$ , respectively.

**Program**

1. Program Euler's method.
2. Program Heun's method.
3. Program fourth order Runge-Kutta method.

Apply all methods, using  $h = 0.1$  to the equations:

$y'(t) = y(t), \quad 0 \leq t \leq 3, \quad y(0) = 1,$  which has the exact solution  $y = e^t$ .

$y'(t) = y(t) - 1, \quad 0 \leq t \leq 3, \quad y(0) = 0,$  which has the exact solution  $y = 1 - e^{-t}$ .

Format the output in a table with following headers:

$t_i$  Euler Error Heun Error RK-4 Error

For the errors use the absolute value of the difference between the approximate solution and the exact solution.

Extra. Try to repeat the above using  $h = 0.1$  for the fourth order Runge-Kutta method,  $h = 0.05$  for Heun's method, and  $h = 0.025$  for Euler's method.