

Math 2650

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Isoclines

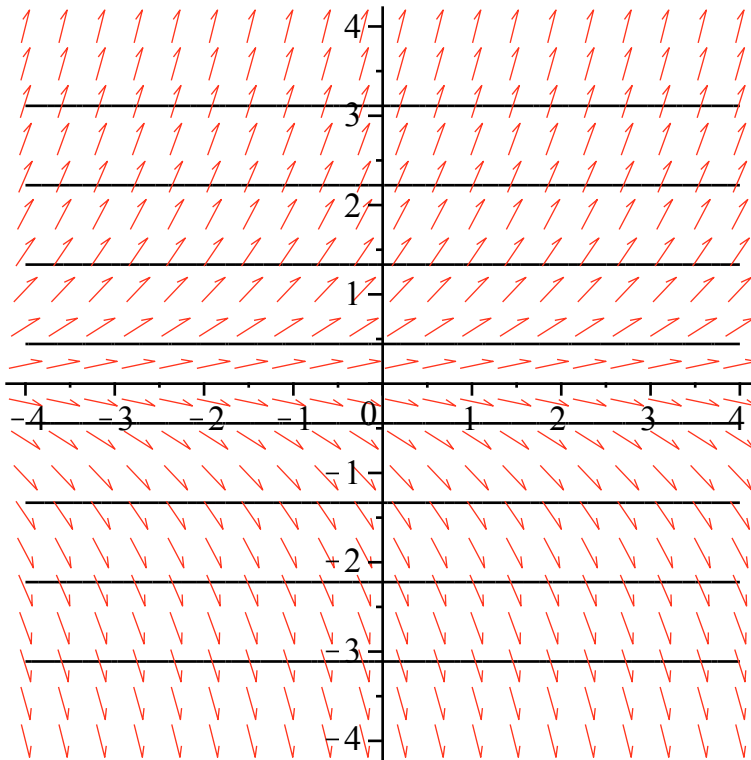
In these examples we illustrate how you can use Maple to draw isoclines (and in fact superpose the isoclines on the direction field). Recall, isoclines are lines along which the slope lines have constant slope (level curves of the derivative).

```
> restart;  
> with(DEtools):  
> with(plots):
```

Consider the equation

$$\frac{dx}{dt} = x$$

```
> DF:=dfieldplot(D(x)(t)=x(t),x(t),t=-4..4,x=-4..4):  
> C:=contourplot(x,t=-4..4,x=-4..4,color=black):  
> display(C,DF);
```



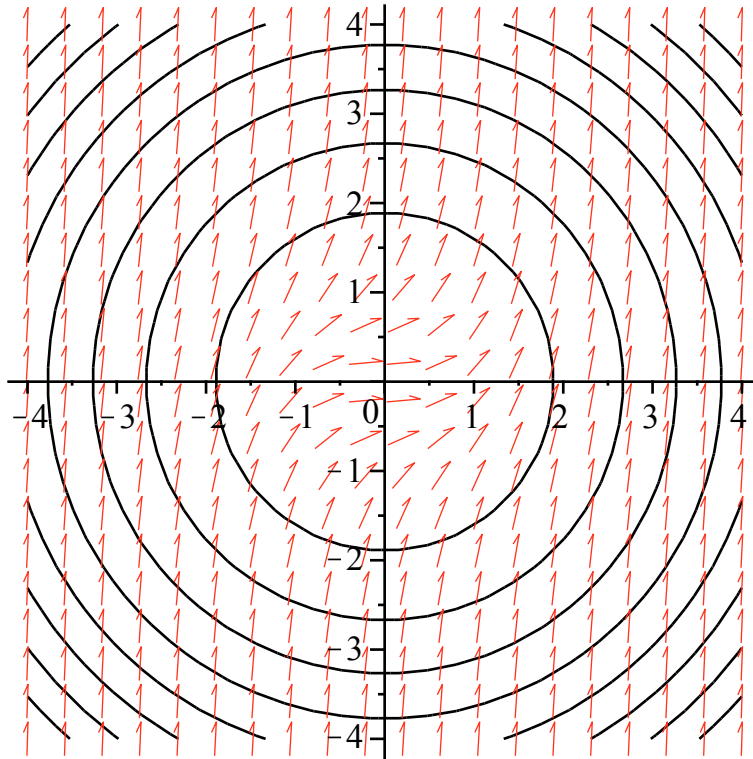
Consider the equation

$$\frac{dx}{dt} = t^2 + x^2$$

```

> DF:=dfieldplot(D(x)(t)=t^2+x(t)^2,x(t),t=-4..4,x=-4..4):
> C:=contourplot(t^2+x(t)^2,t=-4..4,x=-4..4,color=black):
> display(C,DF);

```



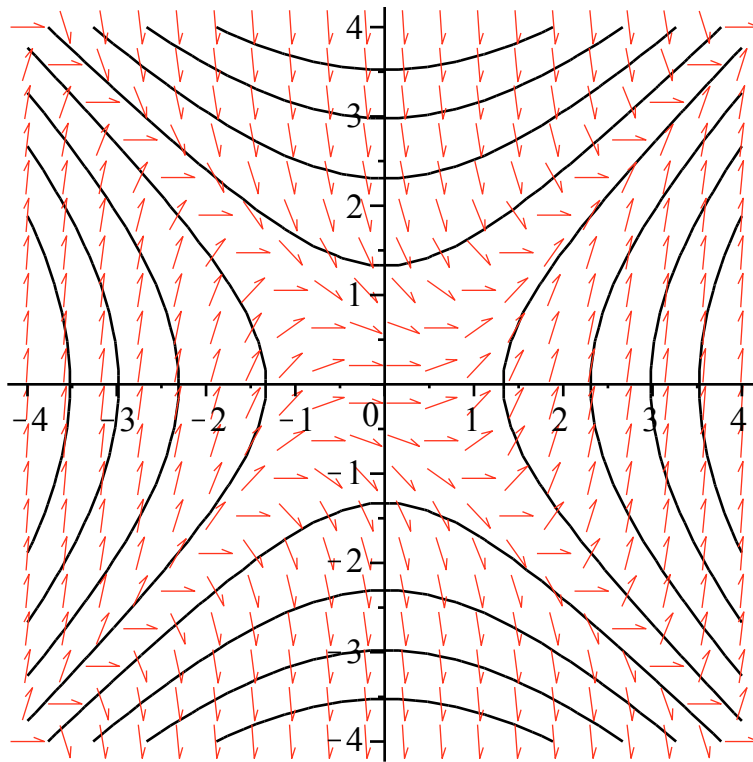
Consider the equation

$$\frac{dx}{dt} = t^2 - x^2$$

```

> DF:=dfieldplot(D(x)(t)=t^2-x(t)^2,x(t),
t=-4..4,x=-4..4):
> C:=contourplot(t^2-x(t)^2,t=-4..4,x=-4..4,
color=black):
> display(C,DF);

```



Trapping regions

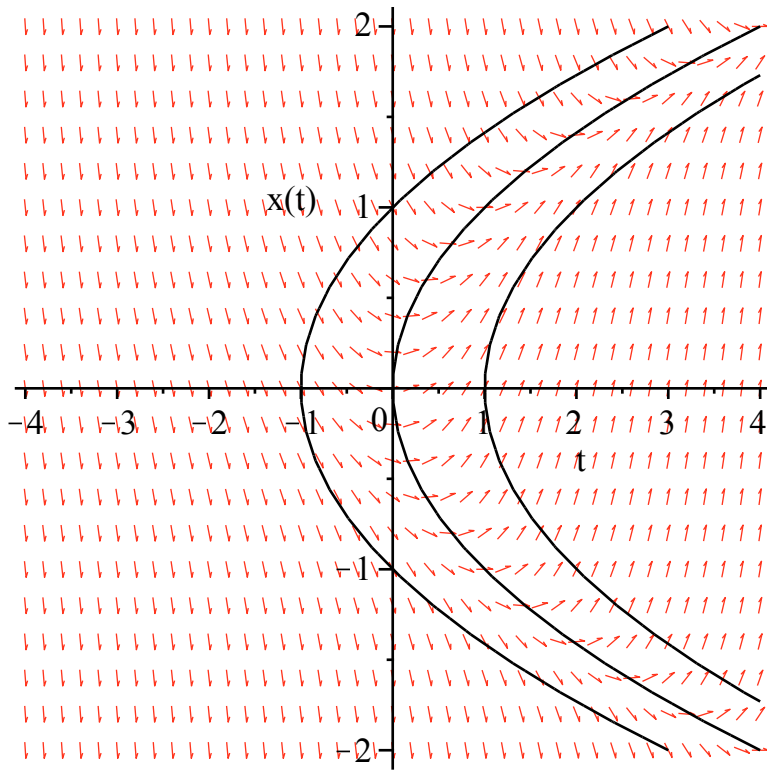
Here we show some examples of trapping regions and how to find them by looking at direction fields.

```
[> restart:with(DEtools):with(plots):
```

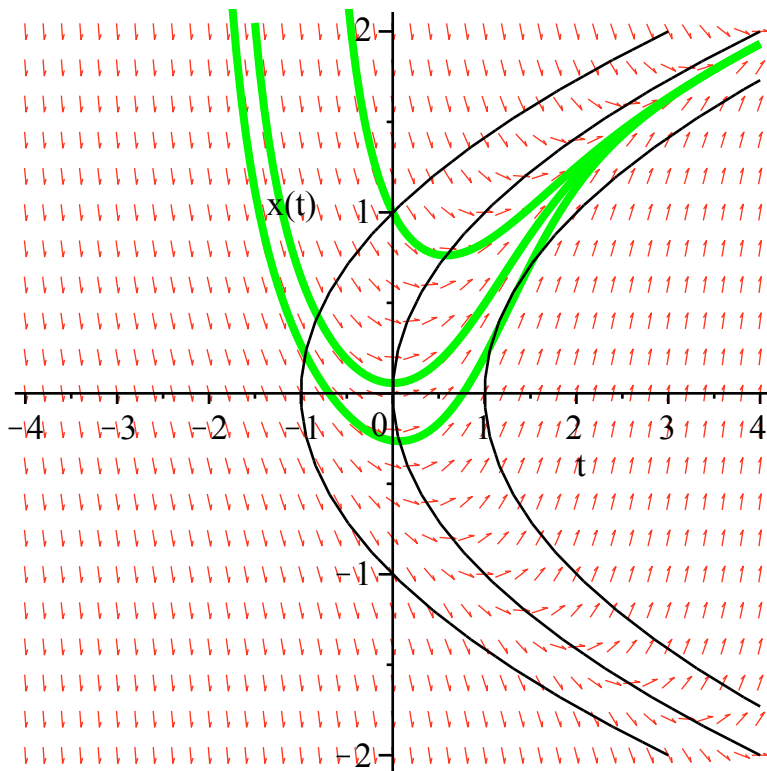
Consider the equation

$$\frac{dx}{dt} = t - x^2$$

```
[> DF:=DEplot(D(x)(t)=t-x(t)^2,x(t),t=-4..4,x=-2..2,
dirgrid=[41,21]):
[> C:=contourplot(t-x^2,t=-4..4,x=-2..2,
contours=[0,1,-1],color=black):
[> display(DF,C);
```



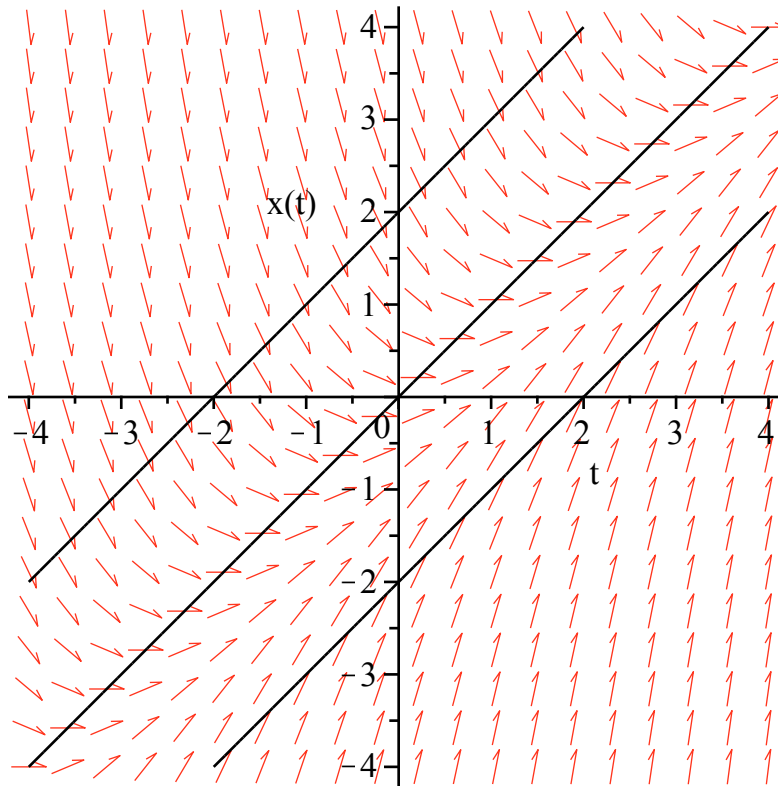
```
> DF1:=DEplot(D(x)(t)=t-x(t)^2,x(t),t=-4..4,
[[x(0)=1],[x(-.3)=-.2],[x(.3)=.1]],x=-2..2,
dirgrid=[41,21],stepsize=.05,linecolor=green):
> display(DF1,C);
```



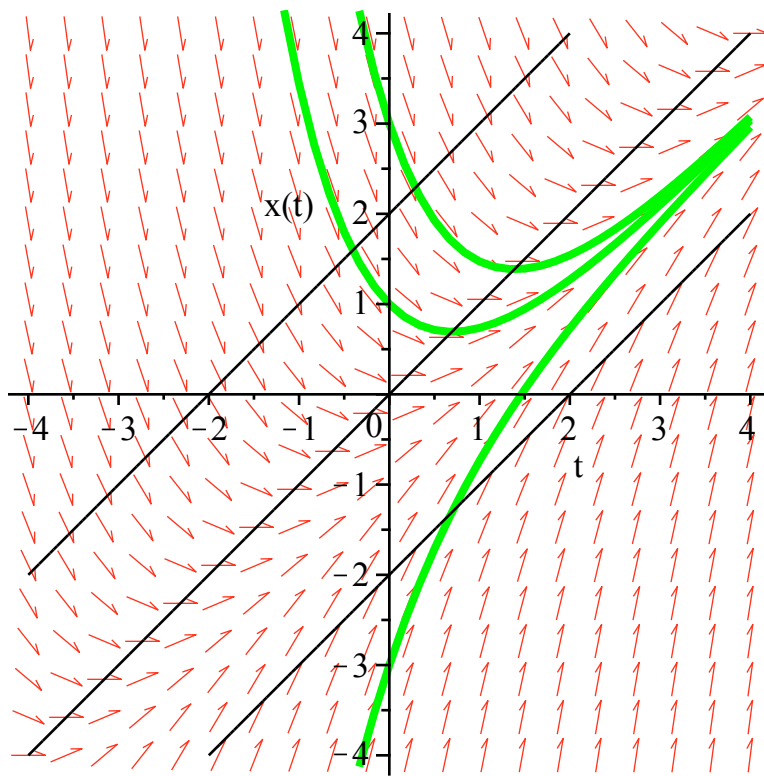
Consider the equation

$$\frac{dx}{dt} = t - x$$

```
> DF:=DEplot(D(x)(t)=t-x(t),x(t),t=-4..4,x=-4..4):  
> C:=contourplot(t-x,t=-4..4,x=-4..4,contours=[-2,0,2],color=  
black):  
> display(DF,C);
```



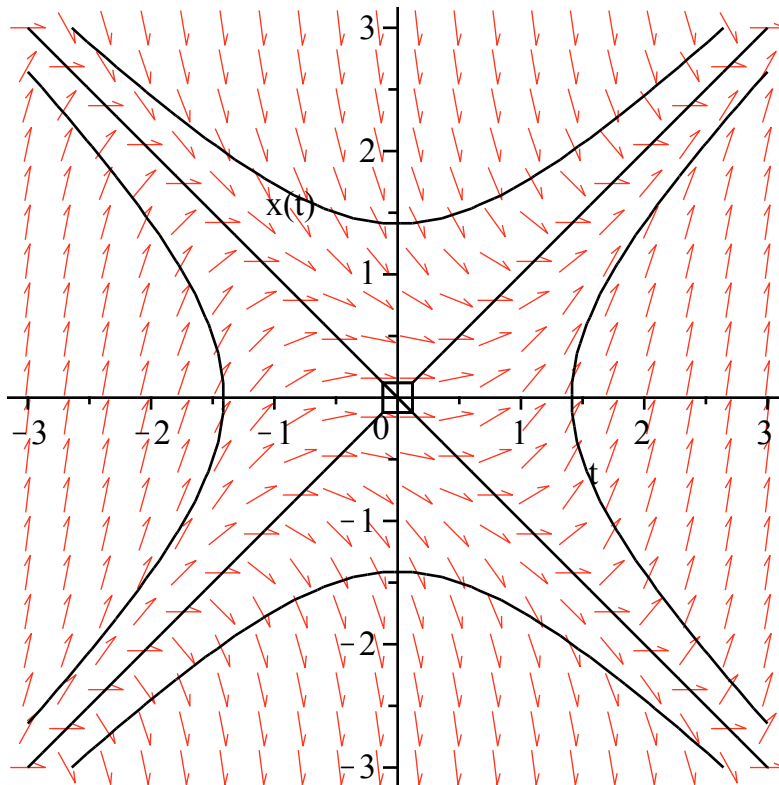
```
> DF1:=DEplot(D(x)(t)=t-x(t),x(t),t=-4..4,x=-4..4,  
[[x(0)=3],[x(0)=1],[x(0)=-3]],linecolor=green):  
> display(DF1,C);
```



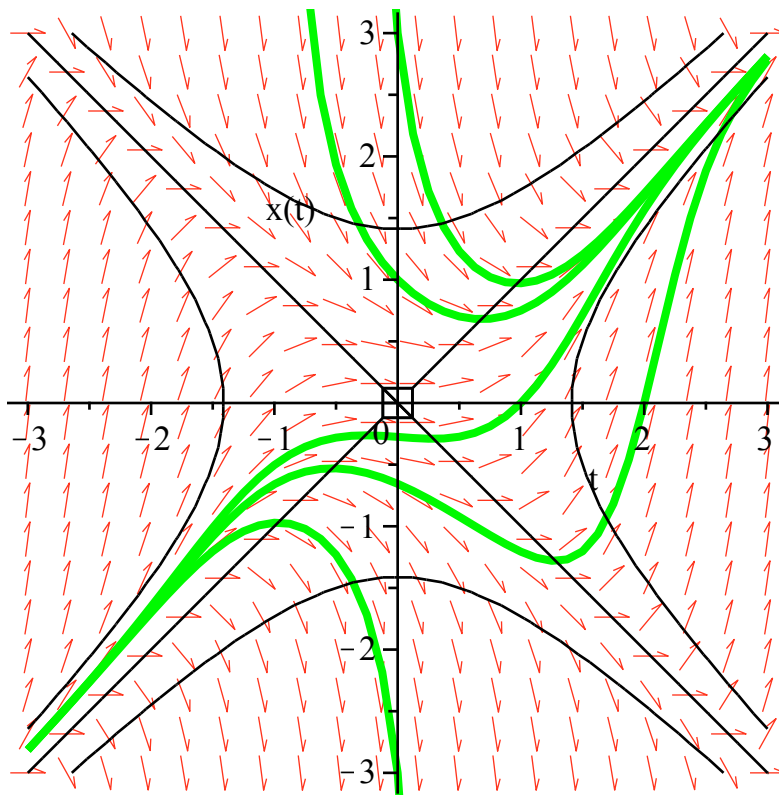
Consider the equation

$$\frac{dx}{dt} = t^2 - x^2$$

```
> DF:=DEplot(D(x)(t)=t^2-x(t)^2,x(t),t=-3..3,x=-3..3):
> C:=contourplot(t^2-x^2,t=-3..3,x=-3..3,
contours=[-2,0,2],color=black):
> display(DF,C);
```



```
> DF1:=DEplot(D(x)(t)=t^2-x(t)^2,x(t),t=-3..3,x=-3..3,
[[x(0)=3],[x(0)=1],[x(0)=-3],[x(1)=0],[x(2)=0]],
linecolor=green):
> display(DF1,C);
```

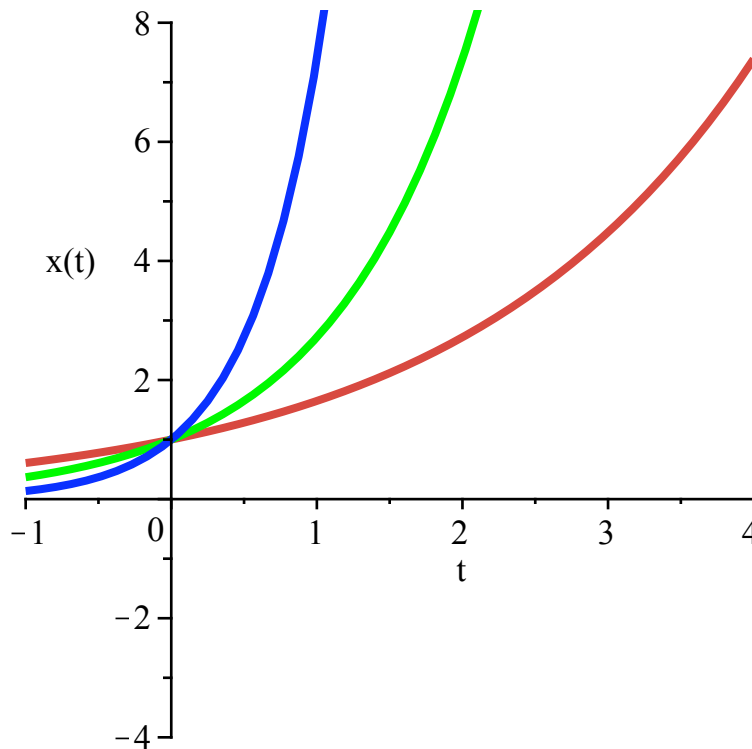


Comparison

Here we show examples of the comparison theorem. Consider the equations

$$\frac{dx}{dt} = \frac{x}{2}, \quad \frac{dx}{dt} = x \quad \text{and} \quad \frac{dx}{dt} = 2x$$

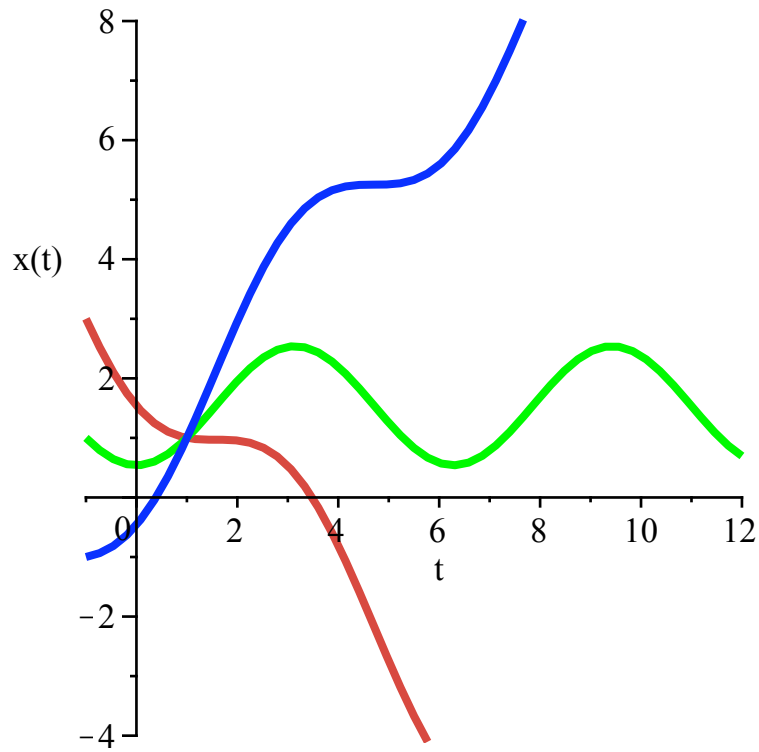
```
> restart:with(DEtools):with(plots):  
> mid:=DEplot(D(x)(t)=x(t),x(t),t=-1..4,x=-4..8,  
  [[x(0)=1]],linecolor=green,arrows=none):  
> big:=DEplot(D(x)(t)=2*x(t),x(t),t=-1..4,x=-4..8,  
  [[x(0)=1]],linecolor=blue,arrows=none):  
> small:=DEplot(D(x)(t)=(1/2)*x(t),x(t),t=-1..4,  
  x=-4..8,[[x(0)=1]],linecolor=orange,arrows=none):  
> display(small,mid,big);
```



Consider the equations

$$\frac{dx}{dt} = \sin(t) - 1, \quad \frac{dx}{dt} = \sin(t) \quad \text{and} \quad \frac{dx}{dt} = \sin(t) + 1$$

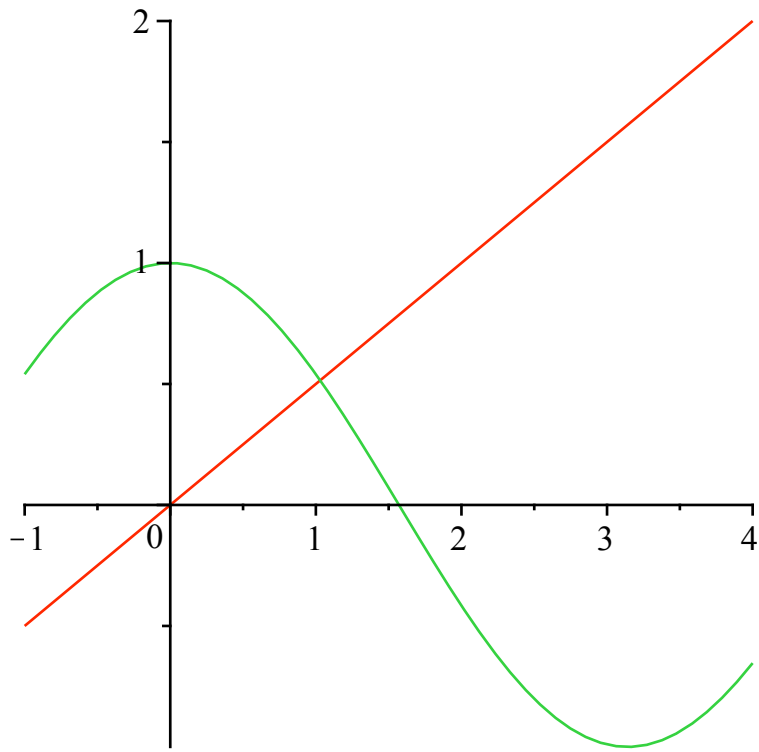
```
> mid:=DEplot(D(x)(t)=sin(t),x(t),t=-1..12,x=-4..8,  
  [[x(1)=1]],linecolor=green,arrows=none):  
> big:=DEplot(D(x)(t)=sin(t)+1,x(t),t=-1..12,x=-4..8,  
  [[x(1)=1]],linecolor=blue,arrows=none):  
> small:=DEplot(D(x)(t)=sin(t)-1,x(t),t=-1..12,x=-4..8,[[x(1)=  
  1]],linecolor=orange,arrows=none):  
> display(small,mid,big);
```



Finally consider the equations

$$\frac{dx}{dt} = \frac{t}{2} \quad \text{and} \quad \frac{dx}{dt} = \cos(t)$$

```
> eq1:=DEplot(D(x)(t)=1/2*t,x(t),t=-1..4,x=-2..4,
[[x(0)=1]],linecolor=green,arrows=none):
> eq2:=DEplot(D(x)(t)=cos(t),x(t),t=-1..4,x=-2..4,
[[x(0)=1]],linecolor=blue,arrows=none):
> plot([1/2*t,cos(t)],t=-1..4);
```



```
> display(eq1,eq2);
```

