A Multi Fluid Analysis of the Ignition Criterion

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Motivation

- The next generation of magnetic confinement nuclear fusion experiments aims to achieve burning plasma conditions.
- A clear understanding of performance requirements needed to obtain burning or ignition conditions is desirable.
- Our knowledge to that purpose has not advanced much since Lawson's original work[1].
- We include additional physics in a zero- and one-dimensional analysis of the plasma to improve our estimate of plasma properties relevant to ignition and burning plasma conditions.

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[1] J. D. Lawson, Proc. Phys. Soc. London Sect. B 70, 6 (1957)

Outline

- Modified ignition criterion:
  - Include two-fluid and α-particle effects.
- Compute and compare $\dot{T}$ vs. T curves for various models.
  - Determine the relevance two-fluid and α-particle effects on the minimum heating needed for ignition.
- Consider one-dimensional, two-parameter density and temperature profiles and evaluate their effect on ignition physics.

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Time-Independent Analysis: In Previous Episodes . . .

The Lawson criterion is derived starting from the single-fluid zero-dimensional energy balance:

$$\frac{E_\alpha}{16} p^2 \frac{< \sigma v >}{T^2} + S_h = \frac{C_B}{4} \frac{p^2}{T^{3/2}} + \frac{3}{2} \frac{p}{\tau_E} \cancel{\left[+ \frac{3}{2} \frac{dp}{dt} \right]}. \qquad (1)$$

A straightforward manipulation gives the ignition criterion

$$p\tau_E = 2nT\tau_E \geq \frac{T^2}{\frac{E_\alpha}{24} < \sigma v > - \frac{C_B}{6} T^{1/2}}. \qquad (2)$$

We now want to see how this is modified by multi-fluid effects.

The Starting Equations Are the Time-Dependent Three-Fluid Energy Conservation Equations.

The starting point is the system of zero-dimensional conservation equations for the three species, ions, electrons and αs:

$$\frac{3}{2}n\frac{\partial T_i}{\partial t} = S_{hi} - \frac{3}{2}\frac{p_i}{\tau_{Ei}} + \frac{3}{2}\frac{n(T_e - T_i)}{\tau_{eq}} \quad (3)$$

$$\frac{3}{2}n\frac{\partial T_e}{\partial t} = S_{he} - \frac{3}{2}\frac{p_e}{\tau_{Ee}} + \frac{n_\alpha}{\tau_\alpha}E_\alpha - C_B\frac{p_e^2}{T_e^{3/2}} + \frac{3}{2}\frac{n(T_i - T_e)}{\tau_{eq}} \quad (4)$$

$$\frac{\partial n_\alpha}{\partial t} = \frac{n^2}{4} < \sigma v > - \frac{n_\alpha}{\tau_\alpha} - \frac{n_\alpha}{\tau_{E\alpha}}. \quad (5)$$

The System Simplifies for Steady-State.

To write an ignition criterion, we focus on the time-independent energy balance:

$$S_{hi} - \frac{3}{2}\frac{p_i}{\tau_{Ei}} + \frac{3}{2}\frac{n(T_e - T_i)}{\tau_{eq}} = 0 \qquad (6)$$

$$S_{he} - \frac{3}{2}\frac{p_e}{\tau_{Ee}} + \frac{n_\alpha}{\tau_\alpha}E_\alpha - C_B\frac{p_e^2}{T_e^{3/2}} + \frac{3}{2}\frac{n(T_i - T_e)}{\tau_{eq}} = 0 \qquad (7)$$

$$\frac{n^2}{4} < \sigma v > - \frac{n_\alpha}{\tau_\alpha} - \frac{n_\alpha}{\tau_{E\alpha}} = 0. \qquad (8)$$

Note that for $\tau_{E\alpha} \rightarrow \infty$ (perfectly confined αs) alpha particles drop out of the system.

An Ideal Multi-Fluid Ignition Criterion Is Written.

- For convenience we assume $\tau_{Ei} = k_1 \hat{\tau}_E$, $\tau_{Ee} = k_2 \hat{\tau}_E$.
- After some straightforward algebra, the ideal steady-state power balance is given by:

$$\frac{n}{4} < \sigma v > \frac{\tau_{E\alpha}}{\tau_{E\alpha} + \tau_\alpha} E_\alpha - C_B n \sqrt{\frac{k_1 \hat{\tau}_E + \tau_{eq}}{k_1 \hat{\tau}_E} T_i} - \frac{3}{2} \left(\frac{2}{\hat{\tau}_E} + \frac{\tau_{eq}}{k_1 k_2 \hat{\tau}_E^2} \right) T_i \geq 0. \quad (9)$$

The associated equation Eq. (9)=0 has analytic roots, but they won't fit in the presentation. . .

- We have used the definition: $\hat{\tau}_E = 2 \frac{\tau_{Ei} \tau_{Ee}}{\tau_{Ei} + \tau_{Ee}}$

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The No-Bremsshtrahlung Ignition Criterion Is Written.

If Bremsstrahlung is neglected, a simple expression is obtained:

$$nT_i\hat{\tau}_E \geq \frac{6T_i^2}{E_\alpha < \sigma v >}\frac{\tau_{E\alpha}+\tau_\alpha}{\tau_{E\alpha}}\left[1+\sqrt{1+\frac{1}{6}\frac{n<\sigma v>E_\alpha}{T_i}\frac{\tau_{E\alpha}\tau_{eq}}{k_1k_2(\tau_{E\alpha}+\tau_\alpha)}}\right]. \qquad (10)$$

Note that, with $C_B = 0$, this reduces to Eq. (2) in the limits

1. $\tau_{eq} \to 0$ and
2. $\tau_{E\alpha} \to \infty$.

Bremsshtrahlung Is Considered Pertubatively.

Assuming $\tau_{eq} \ll \hat{\tau}_E$, Bremsstrahlung can be included to obtain the somewhat unsatisfying relation:

$$nT_i\hat{\tau}_E \geq \frac{6T_i^2 + C_B n^2 \frac{\tau_{eq}}{k_1} T_i^{3/2}}{E_\alpha < \sigma v > \frac{\tau_{E\alpha}}{\tau_{E\alpha}+\tau_\alpha} - 4C_B\sqrt{T_i}} + \frac{\sqrt{36 + \frac{3\tau_{eq}}{k_1}\left(\frac{2n<\sigma v>}{k_2}\frac{\tau_{E\alpha}}{\tau_{E\alpha}+\tau_\alpha}\frac{E_\alpha}{T_i} - c_2 C_B \frac{n}{\sqrt{T_i}}\right)}}{E_\alpha < \sigma v > \frac{\tau_{E\alpha}}{\tau_{E\alpha}+\tau_\alpha} - 4C_B\sqrt{T_i}} T_i^2, \qquad (11)$$

where $c_2 = 2/k_2 - 1 \equiv \tau_{Ee}/\tau_{Ei}$.

- Our conclusion is that multi-fluid effects negatively impact ignition, unless αs are perfectly confined.
- The presence of a finite ion/electron temperature equilibration time further increases the τ_E requirements for ignition.

Time-Dependent Analysis: In Previous Episodes . . .

- The standard approach considers a single fluid and the α power is immediately and entirely delivered to the plasma.
- Linear analysis is used to determine the stability of T:
  - ✓ Positive $\dot{T} \equiv dT/dt$ corresponds to an unstable temperature (temperature will grow if perturbed);
  - ✓ Negative $\dot{T}$ corresponds to a stable temperature (temperature will **not** grow if perturbed).
- $\dot{T}$ is negative for small $T \to$ heating power is needed.

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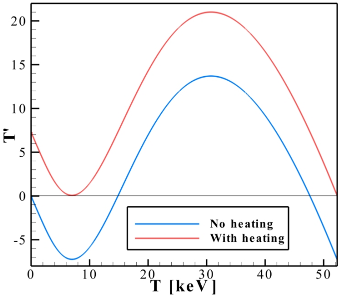
In Previous Episodes . . . [Continued].

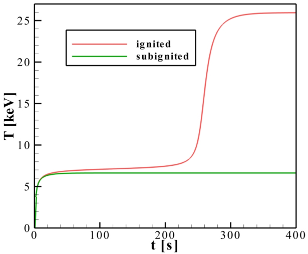
- With no heating, $\dot{T} = 0$ corresponds to ignition points (α power = losses).
- To reach an ignition point from a cold plasma, heating power is needed.
- One may also want some heating power at high temperature for burn control.
- Turning power on and off only shifts the curve up and down.
- $\dot{T} = 0$ points move farther apart with heating on.



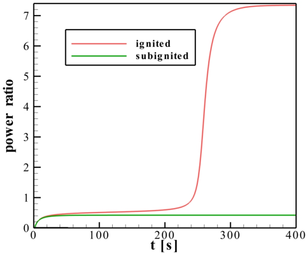
$\dot{T}$ vs. T with and without heating, single fluid model

The Complete Multi-Fluid Model Is Used.

- The time-dependent power balance is reexamined considering ions, electrons and α.
- (Ion) heating power S_{hi} is increased until "ignition", i.e. until T_i "jumps" to a higher value (bifurcation).
- For an "ignited" case, α power > S_{hi}.
- S_{hi} is kept constant during the simulation.
- S_{hi} for the two cases on the right differs by less than 1%.
- $\hat{\tau}_E = 2$s in this page.



T_i vs. t, ignited and sub-ignited case



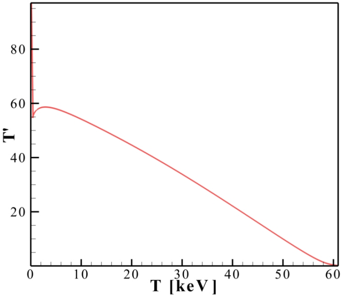
α power / heating power vs. t

Heating Needed for Ignition Can Be Defined In Terms of $\dot{T}$

- The relevant temperature for defining ignition is T_i.
- As for the single-fluid case, the minimum S_{hi} for ignition can be defined as the S_{hi} that makes $\dot{T}_i$ positive "up to large T_i".
- This requires the minimum in $\dot{T}_i$ to be $\gtrsim 0$.
- If $\hat{\tau}_E$ is too small, $\dot{T}_i$ has no minimum ($\hat{\tau}_E = 1$s in the figure on the right).

$\hat{\tau}_E$	1 [s]
τ_{Ee}/τ_{Ei}	1
$\tau_{E\alpha}$	10^6 [s]
n	10^{20} [m^{-3}]

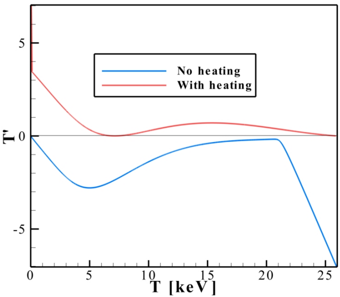
Parameters used in this section



$\dot{T}_i$ vs. T_i, non-igniting

System Nonlinearity Manifests Itself in Calculated $\dot{T}_i$.

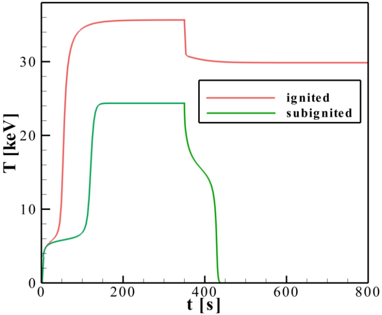
- Contrary to the single-fluid case, the $\dot{T}_i$ curve has a different shape with or without heating.
- When the heating is turned off, the plasma may simply cool down to 0 temperature.
- Since nonlinearity is retained, $\dot{T}_i$ depends on the time history of the system.
- This suggests the question: *how best to define the minimum S_{hi} for ignition?*



$\dot{T}_i$ vs. T_i, non-igniting

Heating for Ignition Is Defined in Terms of $T_i(t \to \infty)$.

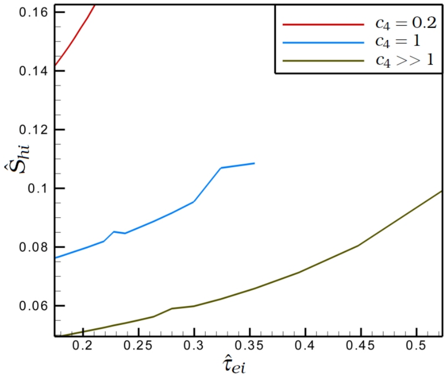
- We **define** S_{hi} to be sufficient for ignition if:
  1. T_i "jumps" to a "high" value;
  2. after the heating is turned off, T_i and P_α remain finite.
- Equivalent definition: S_{hi} is sufficient for ignition if $T_i(t \to \infty) > T_I^{ign}$ (the stable ignition root).
- $\hat{\tau}_E = 3$s in both cases, but $\tau_{E\alpha} = \hat{\tau}_E$ (sub-ignited) and $\tau_{E\alpha} \gg \hat{\tau}_E$ (ignited).
- Heating is turned off at $t = 350$.



Time evolution, ignited and sub-ignited cases.

Heating Power for Ignition Depends on Multi-Fluid Parameters.

- We fix $\tau_{Ei} = 5s$, then vary $\tau_{Ee} \equiv c_2 \tau_{Ei}$ and $\tau_{E\alpha} \equiv c_4 \tau_{Ei}$.
- More heating power S_{hi} is needed with low α or electron confinement.
- Lower plasma energy is reached with poor confinement even for ignited plasmas.
- $\hat{S}_{hi}$ is normalized to the plasma thermal energy.
- $\hat{\tau}_{ei} \equiv \tau_{ei}^{ref} / \hat{\tau}_E$, τ_{ei}^{ref} is kept fixed.



Heating power to reach ignition

One-Dimensional Parameters Are Introduced

- We introduce the density and temperature profiles:

$$n(r,t) = n_0(t)\left(1 - r^{\theta}\right)^{\eta} \qquad T_{i,e}(r,t) = T_{0;i,e}(t)\left(1 - r^{\nu}\right)^{\mu},$$

$$\text{with } 0.1 \le (\mu;\eta) \le 2 \qquad \text{and } 1.1 \le (\nu;\theta) \le 4.$$

- Spatial profiles are fixed in time even during time-dependent simulations: We assume that profile equilibration is faster than transients (i.e., time evolution of n_0 etc.).
- Ion and electron temperature profiles are kept identical, but could in principle be different. Note that $T_{0;i} \neq T_{0;e}$!

One-Dimensional Problem Setup

- For n_α the "equilibrium" spatial profile is used, obtained from

$$\frac{\partial n_\alpha(r,t)}{\partial t} = \frac{n(r,t)^2}{4} < \sigma v > (r,t) - \frac{n_\alpha(r,t)}{\tau_\alpha(r,t)} - \frac{n_\alpha(r,t)}{\tau_{E\alpha}} \quad (12)$$

  and normalized to 1 at $r = 0$.
- Keep in mind that $< \sigma v > = < \sigma v > (T_i(r,t,))$ and $\tau_\alpha = \tau_\alpha(n(r,t), T_e(r,t,))$.
- The ion-electron equilibration time τ_{eq} also depends on profiles, but energy confinement times τ_{Ei}, τ_{Ee}, $\tau_{E\alpha}$ are entered as constant values for each case.

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One-Dimensional Problem Setup [2]

- The full set of equations:

$$\frac{3}{2}n(r,t)\frac{\partial T_i(r,t)}{\partial t} = S_{hi} - \frac{3}{2}\frac{n(r,t)T_i(r,t)}{\tau_{Ei}} + \frac{3}{2}\frac{n(r,t)\left(T_e(r,t)-T_i(r,t)\right)}{\tau_{eq}}, \tag{13}$$

$$\begin{aligned}\frac{3}{2}n(r,t)\frac{\partial T_e(r,t)}{\partial t} &= S_{he} - \frac{3}{2}\frac{n(r,t)T_e(r,t)}{\tau_{Ee}} + \frac{n_\alpha(r,t)}{\tau_\alpha}E_\alpha \\ &- C_B\frac{\left(n(r,t)T_e^2(r,t)\right)}{T_e^{3/2}(r,t)} + \frac{3}{2}\frac{n(r,t)\left(T_i(r,t)-T_e(r,t)\right)}{\tau_{eq}},\end{aligned} \tag{14}$$

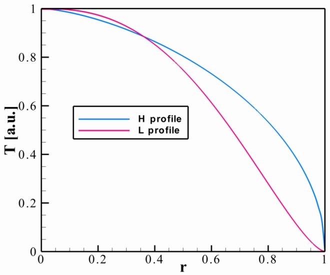
$$\frac{\partial n_\alpha(r,t)}{\partial t} = \frac{n^2(r,t)}{4} < \sigma v > - \frac{n_\alpha(r,t)}{\tau_\alpha} - \frac{n_\alpha(r,t)}{\tau_{E\alpha}} \tag{15}$$

  is first integrated (i.e., averaged) in space at each time step, then advanced in time.

- Note that $n'_\alpha(t) \neq 0$ since only the shape (and not the numerical value) of n_α is determined from Eq. (12).

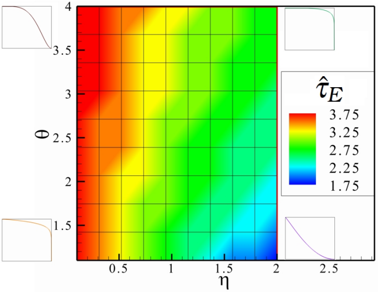
The importance of Profiles Is Studied

- The 1D profile definitions allow in principle for a 4D (η, θ, μ, ν) space to be explored for profile optimization (6D if one allows for different profiles for T_i and T_e).
- In practice, temperature profiles are determined by transport and are less amenable to external control than density profile.
- In most cases, we assign either $T_{i,e}(r) \equiv T_L(r)$ or $T_{i,e}(r) \equiv T_H(r)$ (L- or H-mode-like profiles).

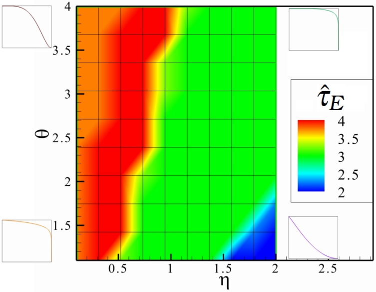


L- $(\mu = 1.5,\ \nu = 2.5)$ and H- $(\mu = 0.5,\ \nu = 1.5)$ mode temperature profiles

Minimum $\hat{\tau}_E$ for Ignition Depends on Profiles.



Minimum $\hat{\tau}_E$ for ignition, L-mode temperature profiles

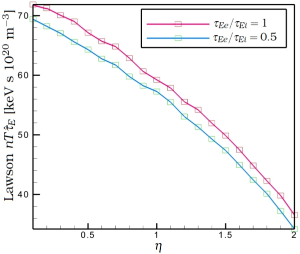


Minimum $\hat{\tau}_E$ for ignition, H-mode temperature profiles

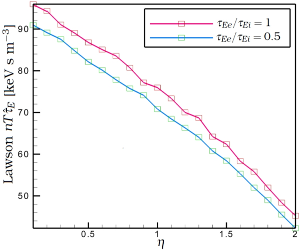
- Density profiles are varied keeping temperature profiles fixed.
- Average n is fixed for all runs.
- The energy confinement time needed for ignition depends on the density and temperature profiles.

The Complete Multi-Fluid Model Is Used.

- Lawson $nT\hat{\tau}_E$ is shown for L- and H-mode temperature $(T \equiv T_i + T_e)$.
- η and θ are varied linearly (from bottom right to top left corner in previous page).
- With poor electron confinement a larger τ_{Ei} is needed.
- However, a smaller $\hat{\tau}_E$ is sufficient for ignition.
- $c_4 = 1$ for all cases (well confined α).
- In 0D, $nT\hat{\tau}_E \simeq 111$ [keV s 10^{20} m^{-3}]



$nT\hat{\tau}_E$ for ignition, L-mode temperature profile



$nT\hat{\tau}_E$ for ignition, H-mode temperature profile

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Conclusions

- Two-fluid and α effects on ignition have been analyzed.
- The energy confinement time needed for ignition depends on multi-fluid physics.
- The heating power needed for ignition also depends on multi-fluid physics.
- One-dimensional temperature and density profiles influence the ignition criterion.

List of Symbols

n	plasma density	n_α	α particle density
T_i	ion temperature	T_e	electron temperature
C_B	Bremsstrahlung coefficient	$< \sigma v >$	Fusion cross section
$\hat{\tau}_E$	"equivalent" energy confinement time	$\tau_{E\alpha}$	α energy confinement time
$k_1 \hat{\tau}_E$	ion energy confinement time	$k_2 \hat{\tau}_E$	electron energy confinement time
$\tau_{eq} = \tau_{eq}(T_i, T_e)$	T_i/T_e equilibration time	$\tau_\alpha = \tau_\alpha(T_e)$	α/T_e equilibration time