Simulations of Sawtooth Oscillations in CTH

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Compact Toroidal Hybrid

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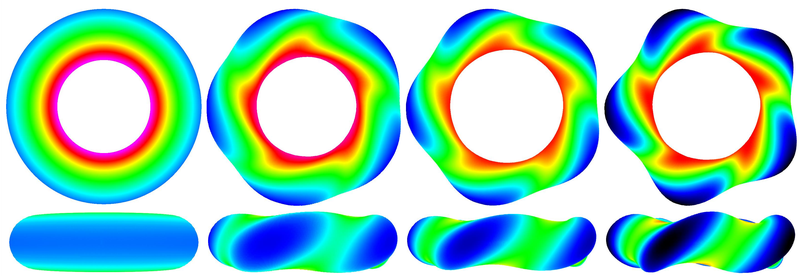
Introduction

- Sawtoothing is sometimes seen in the Compact Toroidal Hybrid (CTH) on SXR signals.
  - CTH is a small stellarator-tokamak hybrid with a field period $N_{fp} = 5, L = 2$ stellarator magnet coil.
- A clear correlation between the sawtooth period and the strength of the helical stellarator field is observed experimentally.
- Past numerical studies of sawtoothing suggests that a resistive MHD model with a highly anisotropic heat flux is adequate for quantitatively recovering many properties of sawteeth in smaller tokamaks with relatively low temperatures.
- Here, we study the effect of three dimensional shaping from the helical stellarator field on sawteeth by evolving extended resistive MHD equations with NIMROD for a sequence of configurations with increasing helical field strength.
- A correlation between sawtooth period and helical field strength is seen in the results.
- Careful attention to numerical convergence was required to obtain the results and these considerations are relevant to simulations of other phenomena in devices with non-axisymmetric plasmas such as perturbed tokamaks, RFPs and stellarators.

Problem Specification

- Extended resistive MHD model
  - Highly anisotropic temperature diffusion
  - Ohmic heat source
  - Careful selection of dissipation, diffusion and source coefficients was required to obtain repeated relaxations that do not decay away.
- Four configurations with increasing amounts of helical stellarator field are considered.
  - Range from $\iota_{vac} = 0$ to 0.097
  - VMEC equilibria are used for initial conditions and magnetic boundary conditions.
- q-profiles increase monotonically in radial direction (tokamak-like profiles).



Temperature isosurfaces for T corresponding to the average value at the last closed flux surface are shown for each configuration. The color indicates the strength of the magnetic field. Helical deformation of the plasma is apparent has ι_{vac} is increased.

Acknowledgements

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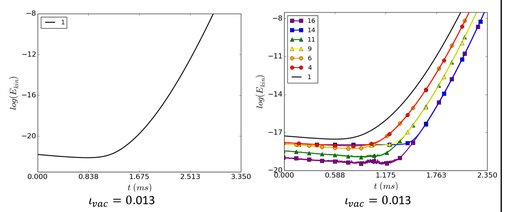
Linear Mode

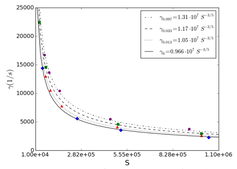
- Unstable mode for axisymmetric case is $n = 1$
- In non-axisymmetric case mode represented with toroidal harmonics $n = 1, N_{fp} \pm 1, N_{fp} \pm 2, N_{fp} \pm 3, \ldots$



$\iota_{vac} = 0.013$ $\iota_{vac} = 0.013$

Magnetic energy of Fourier harmonics involved in representing the linear instability that drives the sawteeth.

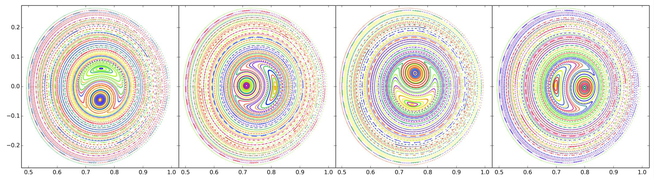
- In all configurations, growth rates for linear mode has tearing scaling with $\gamma \propto S^{-3/5}$
- Constant of proportionality γ_0 increases as ι_{vac} is increased.



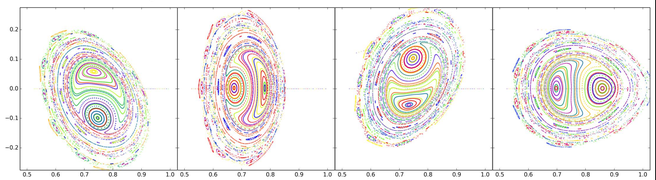
Linear growth rates vs. Lundquist number S for each case. Cases with larger ι_{vac} have larger γ_0

Nonlinear Relaxation

- The non-linear evolution of the sawtooth instability is well described as a Kadomtsev relaxation for all cases.
- The reconnecting plasma core and the growing island are helically deformed as ι_{vac} is increased.
- The displacement of the plasma core has an $n = 1$ variation in direction for all cases.



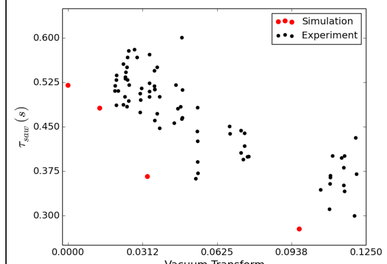
Poincare plots during a relaxation for the $\iota_{vac} = 0$ case at $\phi = 0°, 90°, 180°, 270°$. The growing island and reconnecting core can be seen.



Poincare plots for the $\iota_{vac} = 0.097$ case. The core, island and reconnection layer are helically distorted, but displacement of the core still has a $n = 1$ variation in the direction of radial displacement.

Relaxation Frequency

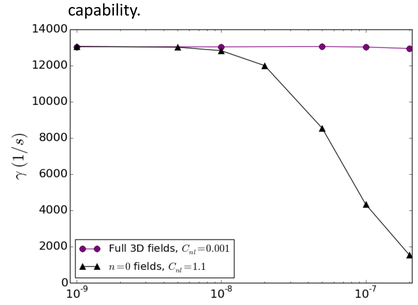
- The sawtooth repetition period τ_{saw} in numerical solutions tends to shorten as ι_{vac} increases.
  - This trend is also observed experimentally.
- May be explained by faster linear growth rates for cases with larger ι_{vac}.



Plots of sawtooth period vs ι_{vac} for the simulations (red) and experimental discharges (black). The loop voltage needed in simulations for constant current is substantially higher than experimental values suggesting resistivity η is higher than in the experimental device. This may at least partially account for the systematic offset between experimental and simulation data.

Temporal Convergence

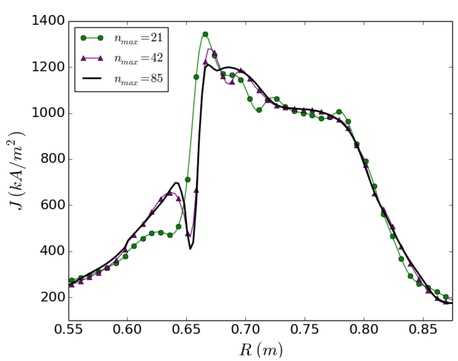
- The semi-implicit operator in NIMROD formerly only used the axisymmetric part of the fields.
  - A large isotropic operator was required for numerical stability in non-axisymmetric cases.
- The capability to use the full 3D fields in the semi-implicit operator has been added to NIMROD.
  - Temporal convergence properties of non-axisymmetric cases is greatly improved by this capability.



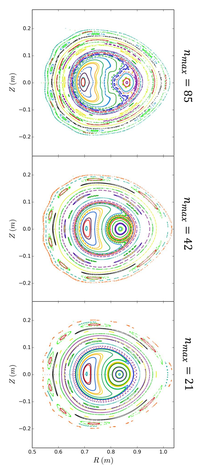
When only the $n = 0$ component of the fields are used in the semi-implicit operator, a large isotropic term is needed for numerical stability. A large isotropic term leads to artificially reduced growth rates for large Δt and a small Δt must be used for convergence.
When the full 3D fields are used in the operator a large isotropic term is not needed.

Spatial Convergence

- The reconnection currently layer should be properly resolved during the relaxations.
- For non-axisymmetric cases, a very high toroidal resolution is required for convergence.
  - NIMROD uses a Fourier spectral representation in the toroidal direction.

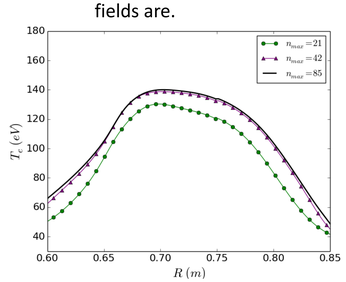


Right: Plots of the current density along a chord passing through the reconnection layer are shown. Under-resolved cases see a Gibbs phenomenon.



Above: Poincare plots during a relaxation for different toroidal resolutions. Under-resolved cases have spurious stochasticity and deformed flux surfaces.

- Highly non-anisotropic temperature diffusion should be spatially resolved for accuracy.
- Non-axisymmetric cases require a very high toroidal resolution for accurate temperature diffusion.
  - May also be affected by how well resolved the equilibrium fields are.



Plots of the temperature along a chord passing through the plasma core. Toroidal resolution is scanned by restarting the simulation with a lower toroidal resolution and running for a set amount of time.