How the Proton Radius Deduced from the Electron-Hydrogen Scattering is Affected by the Flavor Symmetry of Hydrogen Atoms

EUGENE OKS

Physics Department, 380 Duncan Drive, Auburn University, Auburn, AL 36849, USA
Corresponding author email: oksevgu@auburn.edu

ABSTRACT: There is a controversy in measuring r_p persists for over a dozen of years by now – from the time when in 2010 the muonic hydrogen spectroscopy experiment yielded $r_p \approx 0.84$ fm in contrast to the form factor experiment by the Mainz group that produced $r_p \approx 0.88$ fm. Important was that this difference corresponded to about seven standard deviations and therefore was inexplicable. In the intervening dozen of years, more experiments of various kinds were performed in this regard. Nevertheless, the disagreement remains, which is why several different types of new experiments are being prepared for measuring r_p. In one of our previous papers, we pointed out the factor that was never taken into account by the corresponding research community: the *flavor symmetry* of electronic hydrogen atoms, whose existence was confirmed by four different types of atomic/molecular experiments and also evidenced by two different types of astrophysical observations. Specifically, in that paper there was discussed the possible presence of the second flavor of muonic hydrogen atoms (in the corresponding experimental gas) and its effect on the shift of the ground state of muonic hydrogen atoms due to the proton finite size. In the present paper we analyze the effect of the *flavor symmetry* of electronic hydrogen atoms on the corresponding elastic scattering cross-section and on the proton charge radius r_p deduced from the cross-section. As an example, we use our analytical results for reconciling two distinct values of r_p obtained in different elastic scattering experiments: 0.88 fm and 0.84 fm (which is by about 4.5% smaller than 0.88 fm). We show that if the ratio of the second flavor of hydrogen atoms to the usual hydrogen atoms in the experimental gas would be about 0.3, then the extraction of r_p from the corresponding cross-section would yield by about 4.5% smaller value of r_p compared to its true value. We also derive the corresponding general formulas that can be used for interpreting the future electronic and muonic experiments.

Keywords: flavor symmetry of hydrogen atoms; proton radius; electron-hydrogen elastic scattering

1. INTRODUCTION

For a dozen of years there is a controversy concerning the proton charge radius r_p. Namely, in 2010 the muonic hydrogen spectroscopy experiment [1] resulted in $r_p \approx 0.84$ fm in contrast to the form factor experiment by the Mainz group [2] that yielded $r_p \approx 0.88$ fm. This difference was roughly seven standard deviations. Therefore, no explanation existed.

In the subsequent years, more experiments of various kinds were performed in this regard, such as, for example, [3-9]. We also note some of the theoretical papers providing the interpretation or reinterpretation of the experimental results, such as, for instance, [10-12]. More references on the corresponding experimental and theoretical/interpretational papers can be found, for example, in reviews [13-16], as well as in the recent presentations at the 25th European Conference on Few-Body Problems in Physics by Antognini [17], Gao [18], and Meissner [19]. Yet the problem has not been resolved yet and the controversy remains as noted, e.g., in reviews [3, 14, 16].

In paper [20] we pointed out the factor that was never taken into account by the corresponding research community: the *flavor symmetry* of electronic hydrogen atoms, whose existence was confirmed by four different

types of atomic/molecular experiments and also evidenced by two different types of astrophysical observations. (In short, it is about the second solution of the standard Dirac equation for hydrogen atoms, corresponding to the same energy and the first, well-known solution: hence, the additional degeneracy; consequently, an additional conserved quantity; thus, the *flavor symmetry* – more details are provided in Appendix A.) Specifically, in paper [20] there was discussed the possible presence of the second flavor of muonic hydrogen atoms (in the corresponding experimental gas) and its effect on the shift of the ground state of muonic hydrogen atoms due to the proton finite size. It was shown that even a relatively small ratio $\varepsilon \sim 0.1$ of the second flavor and usual muonic hydrogen atoms can lead to about 4% difference in the experimentally deduced parameters.

In the present paper we analyze the effect of the *flavor symmetry* of electronic hydrogen atoms on the corresponding elastic scattering cross-section and on the proton charge radius r_p deduced from the cross-section. As an example, we use our analytical results for reconciling two distinct values of r_p obtained in different experiments on the elastic scattering of electrons on the electronic hydrogen atoms: the value of $r_p = 0.88$ fm from experiments [2, 3, 12] with the value of $r_p = 0.84$ fm from the experiment [8] (which is by about 4.5% smaller than 0.88 fm). Also, we provide the corresponding general formulas that can be used for interpreting the future electronic and muonic experiments.

2. MODEL

In paper [20], based on the analytical results of paper [21], it was shown that if a share ε of the second flavor of *muonic* hydrogen atoms is present in the experimental muonic hydrogen gas, then it affects the shift of the ground state energy ÄE due to the proton finite size in the following way:

$$\Delta E(\varepsilon, R_p) = b(16\beta^{3/2})R_p^2[1/R_p^\beta - \varepsilon R_p/(5\beta) + \varepsilon^2 R_p^2/(100\beta^2)] \tag{1}$$

(here we corrected misprints in the corresponding Eq. (7) from paper [20]). In that equation, ε was the share of the second flavor of muonic atoms in the muonic hydrogen gas and it was considered relatively small ($\varepsilon \ll 1$); R_p was the proton radius in units of the muonic Bohr radius $a_{0\mu} = \hbar^2/(m_\mu e^2)$; $\beta = \alpha^2 \ll 1$ (where α is the fine structure constant); b was a constant of no importance for the purpose of paper [20].

In the present paper we consider how the shift of the ground state energy ΔE of *electronic* hydrogen atoms due to the proton finite size and the corresponding elastic scattering cross-section are affected by the presence of the second flavor of *electronic* hydrogen atoms in the experimental hydrogen gas in the ratio ε to the usual hydrogen atoms. We note that here the restriction $\varepsilon \ll 1$ is not imposed. For the ground state of *electronic* hydrogen atoms, outside the proton, the radial part of the Dirac bispinor, based on Eq. (17) from paper [21], can be represented in the form:

$$f(r) \approx -2\beta^{5/4}\{1/r^{\beta/2} - \varepsilon[R_p^2/(5\beta r^{2-\beta/2})]\}/(1 + \varepsilon^2)^{1/2},$$

$$g(r) \approx 4\beta^{3/4}\{1/r^{\beta/2} - \varepsilon[R_p^2/(10\beta r^{1-\beta/2})]\}/(1 + \varepsilon^2)^{1/2}. \tag{2}$$

Here R_p is the proton "sphere" radius, that is, the boundary between the regular solution of the Dirac equation in the interior region and the singular solution of the Dirac equation in the exterior region. The proportionality relation between R_p and the proton charge radius r_p is specified later on. (In Eq. (2) we corrected a misprint in the expression for g(r) from the corresponding Eq. (2) from paper [20] – in addition to entering the normalizing factor $(1+\varepsilon^2)^{1/2}$, as the denominator.)

After following the same steps as in paper [20], the shift of the ground state energy ÄE due to the proton finite size can be expressed similarly to Eq. (1), but with the denominator $(1 + \varepsilon^2)$ and with the rescaled value of the proton "sphere" radius R_p – namely, now R_p is in units of the *electronic* Bohr radius $a_{0e} = \hbar^2/(m_e e^2)$:

$$\Delta E(\varepsilon, R_p) = const\ R_p^2[1/R_p^\beta - \varepsilon R_p/(5\beta) + \varepsilon^2 R_p^2/(100\beta^{2+\beta})]/(1 + \varepsilon^2). \tag{3}$$

In Eq. (3), the energy is measured in atomic units ($\hbar = m_e = e = 1$).

Next, we estimate the relation between the relative change $\delta\sigma = \Delta\sigma/\sigma$ of the elastic scattering cross-section (for the electron-hydrogen scattering) and the relative shift $\delta E = \Delta E/E$ of the ground state energy caused by the admixture of the second flavor of hydrogen atoms. For our simple model, intended just for getting the message across, for the elastic cross section at the limit of relatively low values of the momentum transfer – the limit relevant to the determination of the proton radius from the electron-hydrogen scattering – we use the relation

$$\sigma = \text{const } (<r^2>)^2, \tag{4}$$

corresponding to Eq. (115.4) from the textbook [22]. In Eq. (4), r is the distance of the atomic electron from the proton; the symbol $<...>$ stands for "averaged".

The unperturbed binding energy of the atomic electron (i.e., for $\varepsilon = 0$) in the ground state E_b (or in any state) of hydrogen atoms is inversely proportional to $(<r^2>)$:

$$E_b = \text{const}/(<r^2>), \tag{5}$$

so that

$$(<r^2>) = \text{const}/E_b. \tag{6}$$

Consequently,

$$|\delta(<r^2>)| = |\delta E_b|, \tag{7}$$

where

$$\delta(<r^2>) = \Delta(<r^2>)/(<r^2>), \qquad \delta E_b = \Delta E_b/E_b. \tag{8}$$

From Eq. (4) it follows that

$$\delta\sigma = \Delta\sigma/\sigma = 2\ \delta(<r^2>), \tag{9}$$

where σ is the corresponding cross-section at $\varepsilon = 0$. Then combining Eq. (9) with Eq. (7) we obtain:

$$|\delta\sigma(\varepsilon, R_p)| = 2|\delta E_b(\varepsilon, R_p)|. \tag{10}$$

Since the unperturbed cross-section σ and the unperturbed binding energy E_b do not depend on ε, then the change of the cross-section $\Delta\sigma$ has the same dependence on ε as ΔE from Eq. (3) (apart from a constant)

$$\Delta\sigma(\varepsilon, R_p) = \text{const } R_p^2[1/R_p^\beta - \varepsilon R_p/(5\beta) + \varepsilon^2 R_p^{2+\beta}/(100\beta^2)]/(1 + \varepsilon^2). \tag{11}$$

Some electron scattering experiments yielded the proton charge radius $r_p = 0.88$ fm [2, 3, 12], while another electron scattering experiment yielded $r_p = 0.84$ fm [8], that is, by about 4.5% less than 0.88 fm. Therefore, we will seek the value of ε, such that

$$\Delta\sigma(\varepsilon, R_p) = \Delta\sigma(0, 0.955R_p). \tag{12}$$

In other words, the purpose of solving Eq. (12) is to show that from the same experimental cross-section, one can find either the smaller value of R_p while neglecting a possible admixture of the second flavor of hydrogen atoms to the experimental hydrogen gas (i.e., at $\varepsilon = 0$) or by 4.5% larger value of R_p at some finite value of ε.

Equation (12) is quadratic with respect to ε. It has the following solutions:

$$\varepsilon_1 = [7.50\text{x}10^7 R_p - (1.284 + 5.14\text{x}10^{15} R_p^2)^{1/2}]/(1.408\text{x}10^{11} R_p^2 - 3.65\text{x}10^4), \tag{13}$$

$$\varepsilon_2 = [7.50\text{x}10^7 R_p + (1.284 + 5.14\text{x}10^{15} R_p^2)^{1/2}]/(1.408\text{x}10^{11} R_p^2 - 3.65\text{x}10^4). \tag{14}$$

For numerically estimating the share of the second flavor of hydrogen atoms in the experimental hydrogen gas, we use for the proton charge radius r_p the value of 0.86 fm, which is the mean value between 0.88 fm and 0.86 fm. After the translation into the atomic units, we get $r_p = 0.0000163$.

The proton "sphere" radius R_p would be by the factor of $(5/3)^{1/2}$ greater than r_p (it would be equal to 0.0000210) if the proton would be a uniformly charged sphere (what the proton is not). The actual value of R_p should be between 0.0000163 and 0.0000210. For further numerical estimates of ε_1 and ε_2 we adopt the value $R_p \approx 0.000018$, so that

$$\varepsilon_1 \approx 0.27, \qquad\qquad \varepsilon_2 \approx -0.36. \qquad\qquad (15)$$

Obviously, the negative value of ε_2 is non-physical.

Due to the proportionality between the proton charge radius r_p and the proton "sphere" radius R_p, the above qualitative and quantitative result for the dependence of R_p, deduced from the elastic cross-section, on the share of the second flavor of hydrogen atoms in the experimental hydrogen gas, is also the same for r_p. In other words, about 30% ratio of the second flavor of hydrogen atoms to the usual hydrogen atoms in the experimental gas can precipitate the conclusion that r_p is by 4.5% smaller than its true value.

For interpreting future experiments on the elastic scattering of electrons or muons on hydrogen atoms, we consider below the general equation

$$\Delta\sigma(\varepsilon, R_p) = \Delta\sigma[0, (1-a)R_p], \qquad\qquad (16)$$

where a is the relative discrepancy between the values of the proton charge radius deduced from different experiments. (For example, in Eq. (12), a was entered as 0.045.) The solutions of the quadratic equation (16) are as follows:

$$\varepsilon_1 = \{[1.408 \times 10^7 R_p^2 + 4a(2-a)(1-2a-a^2-3.52 \times 10^6 R_p^2)]^{1/2} - 3.75 \times 10^3 R_p\}/[2(1-2a-a^2-3.52 \times 10^6 R_p^2)], \qquad (17)$$

$$\varepsilon_2 = \{-[1.408 \times 10^7 R_p^2 + 4a(2-a)(1-2a-a^2-3.52 \times 10^6 R_p^2)]^{1/2} - 3.75 \times 10^3 R_p\}/[2(1-2a-a^2-3.52 \times 10^6 R_p^2)].$$

Only the solution ε_1 is physically admissible: the solution ε_2 is negative and thus physically inadmissible.

2. CONCLUSIONS

We put forward a model showing how the *flavor symmetry* of hydrogen atoms influences the value of the proton charge radius r_p deduced from the experimental results on the elastic scattering of electrons. We provided the corresponding general formulas that can be used for interpreting the future electronic and muonic experiments.

As an example, we applied our analytical results for reconciling two distinct values of r_p obtained in different experiments on the elastic scattering of electrons on the electronic hydrogen atoms: the value of $r_p = 0.88$ fm from experiments [2, 3, 12] with the value of $r_p = 0.84$ fm from the experiment [8] (which is by about 4.5% smaller than 0.88 fm). We demonstrated that if the ratio of the second flavor of hydrogen atoms to the usual hydrogen atoms in the experimental gas would be about 0.3, then the extraction of r_p from the corresponding cross-section would yield by about 4.5% smaller value of r_p compared to its true value.

We do not imply that this simple model is the final resolution of the ambiguity. The intent of our paper is to stimulate further theoretical/interpretational works in this fundamental research field – especially in view of the planned scattering experiments, such as, e.g., MUSE [23], PRad-II [24, 25], COMPASS++/AMBER [26], and ULQ2 [27].

Appendix A. Brief overview of the atomic/molecular experiments and astrophysical observations proving the existence of the flavor symmetry of hydrogen atoms

The standard Dirac equation for hydrogen atoms has two analytical solutions: 1) the weakly singular at small r (called "regular"); 2) the strongly singular at small r (called "singular"). The 2nd solution is usually rejected because the normalization integral diverges at small r. Even after allowing for the finite size of the proton, but modeling the charge distribution inside the proton as that of a uniformly charged sphere or of a uniform spherical shell, the singular solution outside the proton did not work.

In paper [21] there was derived a general class of potentials inside the nucleus, for which the singular solution outside the nucleus can be actually matched with the corresponding regular solution inside the nucleus. This class of potentials included, in particular, those corresponding to the charge distributions that have a peak at r = 0. In the well-known experiments on the elastic scattering of electrons on protons (see, e.g., paper [28] and book [29]), it was found that the charge distribution inside protons does have a peak at r = 0. In papers [21, 30] it was shown that for the actual charge distribution inside the proton, the regular interior solution can be matched with the singular exterior solution for any *l*=0 state of the discrete and continuous spectrum: the singular solution of the standard Dirac equation for hydrogen atoms is legitimate for all S-states.

Both the regular and singular solutions of the Dirac equation outside the proton correspond to *the same energy*. Since this means *the additional degeneracy*, then according to the fundamental theorem of quantum mechanics, there should be an *additional conserved quantity*. Consequently, hydrogen atoms have two flavors, differing by the eigenvalue of this additional, new conserved quantity: hydrogen atoms have *flavor symmetry* [31]. It is called so by analogy with the flavor symmetry of quarks. This is why the second type of hydrogen atoms having only the S-states was called the Second Flavor of Hydrogen Atoms (SFHA).

The most important feature of the SFHA is that due to having only the S-states, from the well-known selection rules of quantum mechanics follows that the *SFHA do not emit or absorb the electromagnetic radiation* (with the exception of the 21 cm line, corresponding to the spin-flip transition between the two hyperfine sublevels of the ground state) – *they remain dark*. The SFHA do not couple not only to the dipole radiation, but also to the quadrupole, octupole, and all higher multipoles. Also, the SFHA cannot exhibit multi-photon transitions.

By now the existence of the SFHA is proven by the following four different types of atomic or molecular experiments.

1. Experimental High-energy Tail of the Linear Momentum Distribution (HTLMD) in the ground state of hydrogen atoms.

The HTLMD, determined by analyzing atomic experiments for a large set of different collisional processes between hydrogen atoms and protons or electrons, was found to fall off much more slowly [32] than the theoretical HTLMD [33]. *The discrepancy was up to 3 or 4 orders of magnitude*.

In paper [21] it was shown that with the allowance for the SFHA, this huge multi-order discrepancy got completely eliminated. This is because for the singular solution outside the proton, a much more rapid increase of the coordinate wave function toward the proton at small r translates into a *much slower fall-off* of the wave function in the p-representation for large p – according to the properties of the Fourier transform. No alternative explanation of the above huge discrepancy was ever provided.

2. Experiments on the electron impact excitation of hydrogen *atoms*

The comparison of the experimental [34] and theoretical [35] ratio of the cross-section σ_{2s} of the excitation of the state 2s to the cross-section σ_{2p} of the excitation of the state 2p, showed that the theoretical ratio was systematically higher than the experimental ratio *by about 20%* - far beyond the experimental error margins of 9%.

The experimental cross-section σ_{2s} was deduced by using the quenching technique: by applying an electric field that mixed the state 2s with the state 2p and then observing the emission of the Lyman-alpha line from the state 2p to the ground state. Here is the central point. In the mixture of the SFHA with the usual hydrogen atoms, both flavors

can be excited to the 2s state. However, the mixing of the 2s and 2p states by the electric field (followed by the emission of the Lyman-alpha line) happens only for the usual hydrogen atoms: the SFHA, due to having only the S-states, cannot not contribute to the observed emission of the Lyman-alpha line.

Therefore, measurements of the cross-section σ_{2s} in this way, should underestimate this cross-section compared to its actual value, while the cross-section σ_{2p} would not be influenced by the presence of the SFHA. In paper [36] it was shown that the above can be eliminated if the SFHA were present in the share ~ 40% in the experimental hydrogen gas. Again, no alternative explanation was ever provided.

3. Experiments on the electron impact excitation of hydrogen *molecules*

For the excitation of the first two stable excited electronic triplet states of H_2 – the state $c^3\Pi_u$ and the state $a^3\Sigma_g^+$ – even the most advanced calculations by the convergent close-coupling method with the total number of states equal to 491 [37] underestimate the experimental cross-sections ([38, 39] by *at least a factor of two*.

In paper [40] it was demonstrated that if in some hydrogen molecules one or both atoms would be the SFHA, then the above very large discrepancy would be eliminated. The reason is the following. For such nontrivial H_2 molecules, the corresponding theoretical cross-section is by a factor of three greater than for the usual H_2 molecules. The presence of about 30% of the SFHA-based H_2 molecules in the experimental gas would suffice for eliminating the above discrepancy. Again, no alternative explanation was ever provided.

4. Experiments on the charge exchange between hydrogen atoms and low energy protons

The experimental cross-sections [41] were noticeably greater than the theoretical ones calculated in paper [42]. Again, this discrepancy can be eliminated if the SFHA was present in the experimental gas [43]. Here is why.

The cross-section for the resonant charge exchange is (roughly) inversely proportional to the square of the ionization potential U_{ioniz} from the particular atomic state. For the usual hydrogen atoms, U_{ioniz} increases due to the Stark shift by the field of the incoming proton.

However, the energy levels of the SFHA do not shift in the electric field. Once again, no alternative explanation was ever provided.

There are also *two kinds of the astrophysical evidence* of the existence of the SFHA.

1. Perplexing observation of the redshifted 21 cm spectral line from the early Universe

The observed absorption in this spectral line was found to be *two times stronger* than predicted by the standard cosmology [44]. The consequence of this large discrepancy was that *the gas temperature* of the hydrogen clouds *was in reality significantly smaller* than predicted by the standard cosmology.

Barkana [45] theorized that some unspecified dark matter collided with the hydrogen gas and made it cooler compared to the standard cosmology. He estimated that for fitting the observations, the mass of these dark matter particles should be of the same order as protons, or neutrons, or hydrogen atoms. In paper [30] the following scenario was analyzed: what if Barkana's unspecified dark matter particles are the SFHA?

The SFHA do not couple to the electromagnetic radiation except for the radiative transitions between the two hyperfine sublevels of the ground state corresponding to the same 21 cm wavelength as for usual hydrogen atoms. In paper [30] it was expounded that in the course of the Universe expansion, the SFHA (due to having only S-states) decouple from the cosmic microwave background radiation earlier than the usual hydrogen atoms. Therefore, the SFHA cool down more rapidly than the usual hydrogen atoms (the latter decoupling from the cosmic microwave background radiation significantly later). Therefore, the spin temperature, controlling the intensity of the absorption signal in the 21 cm line, was lower for the SFHA. In paper [30] it was demonstrated that this explains the observed anomalous absorption in the 21 cm line both *qualitatively and quantitatively*.

2. Observed anomalous distribution of dark matter in the Universe

It was found to be smoother, less clumpy than predicted by Einstein's general relativity [46], what caused calls for

new physical laws. However, in paper [47] it was demonstrated that this perplexing observation can be also explained *qualitatively and quantitatively* by using the SFHA – without resorting to new physical laws.

The above SFHA-based explanations of these two puzzling astrophysical observations made the SFHA one of the leading candidates for dark matter or for a part of it – see, e.g., review [48].

References

1. Pohl, R. et al. The Size of the Proton. *Nature* ***2010****, 466*, 213-216.

2. Bernauer, J.C. et al. High-Precision Determination of the Electric and Magnetic Form Factors of the Proton. *Phys. Rev. Lett.* **2010**, *105*, 242001.

3. Zhan, X. et al. High-Precision Measurement of the Proton Elastic Form Factor Ratio ì$_p$G$_E$/G$_M$ at Low Q^2. *Phys. Lett. B* **2011**, *705*, 59-64.

4. Antognini, A. et al. Proton structure from the measurement of 2S-2P transition frequencies of muonic hydrogen. *Science* **2013**, *339*, 417-420.

5. Mihovilovic, M. et al. First Measurement of Proton's Charge Form Factor at Very Low Q^2 with Initial State Radiation. *Phys. Lett. B* **2017**, *771*, 194-198.

6. Fleurbaey, H. et al. New Measurement of the 1S - 3S Transition Frequency of Hydrogen: Contribution to the Proton Charge Radius Puzzle. *Phys. Rev. Lett.* **2018**, *120*, 183001.

7. Bezginov, N. et al. A measurement of the atomic hydrogen Lamb shift and the proton charge radius. *Science* **2019**, *365*, 1007–1012.

8. Xiong, W. et al. A small proton charge radius from an electron–proton scattering experiment. *Nature* **2019**, *575*, 147–151.

9. Brandt, A.D.; Cooper, S.F.; Rasor, C.; Burkley, Z.; Matveev, A.; Yost, D.C. Measurement of the 2S$_{1/2}$ − 8D$_{5/2}$ Transition in Hydrogen. *Phys. Rev. Lett.* **2022**, *128*, 023001.

10. Sick, I. Problems with Proton Radii. *Progr. Part. Nucl. Phys.* **2012**, *67*, 473-478.

11. Lee, G.; Arrington, J.R.; Hill, R.J. Extraction of the Proton Radius from Electron-Proton Scattering Data. *Phys. Rev. D* **2015**, *92*, 013013.

12. Mihovilovic, M. et al. The Proton Charge Radius Extracted from the Initial-State Radiation Experiment at MAMI. *Eur. Phys. J. A* **2021**, *57*, 107.

13. Karr, J.P.; Marchand, D. Progress on the proton radius puzzle. *Nature* **2019**, *575*, 61–62.

14. Bernauer, J.C. The Proton Radius Puzzle – 9 Years Later. *EPJ Web of Conferences* **2020**, *234*, 01001.

15. Pacetti, S.; Tomasi-Gustafsson, E. The Origin of the Proton Radius Puzzle. *Eur. Phys. J. A* **2021**, *57*, 72.

16. Gao, H.; Vanderhaeghen, M. The Proton Charge Radius. *Rev. Mod. Phys.* **2022**, *94*, 015002.

17. Antognini, A. Precision Benchmarks for Nuclear and Atomic Physics from Laser Spectroscopy of Muonic Atoms. *The 25th European Conference on Few-Body Problems in Physics* **2023** https://indico.him.uni-mainz.de/event/150/contributions/1128/

18. Gao, H. Recent Results on Proton Charge Radius and Polarizabilities. *The 25th European Conference on Few-Body Problems in Physics* **2023** https://indico.him.uni-mainz.de/event/150/contributions/1135/

19. Meißner, U.-G. The Proton Radius and its Relatives A.D. 2023. *The 25th European Conference on Few-Body Problems in Physics* **2023** https://indico.him.uni-mainz.de/event/150/contributions/1148/

20. Oks, E. A Possible Explanation of the Proton Radius Puzzle based on the Second Flavor of Muonic Hydrogen Atoms. *Foundations* **2022**, *2*, 912-917.

21. Oks, E. High-Energy Tail of the Linear Momentum Distribution in the Ground State of Hydrogen Atoms or Hydrogen-like Ions", *J. Phys. B: At. Mol. Opt. Phys.* **2001**, *34*, 2235-2243.

22. Landau, L.D.; Lifshitz, E.M. *Quantum Mechanics;* Pergamon: Oxford, 1965.

23. Kohl, M. Lepton Universality Test with MUSE at PSI. *J. Phys. Conf. Series* **2022**, *2391*, 012015.

24. Gasparian, A. et al. PRad-II: A New Upgraded High Precision Measurement of the Proton Charge Radius. **2020** https://arxiv.org/abs/2009.10510

25. Lin, Y.-H.; Hammer, H.-W.; Meißner, U.-G. Differential Cross Section Predictions for PRad-II from Dispersion Theory. *Phys. Lett. B* **2022**, *827*, 136981.

26. Adams, B. et al. Letter of Intent: A New QCD Facility at the M2 Beam Line of the CERN SPS COMPASS++/AMBER. **2019** https://arxiv.org/abs/1808.00848

27. Suda, T. Low-Energy Electron Scattering Facilities in Japan. J. Phys. Conf. Series **2022**, *2391*, 012004.

28. Simon, G.; Schmitt, Ch.; Borkowski, F.;Walther, V.H. Absolute electron-proton cross sections at low momentum transfer measured with a high pressure gas target system. *Nucl. Phys.* **1980**, *A333*, 381.

29. Perkins, D.H. *Introduction to High Energy Physics*; Addison-Wesley: Menlo Park (CA), 1987, Sect. 6.5.

30. Oks, E. Alternative Kind of Hydrogen Atoms as a Possible Explanation of the Latest Puzzling Observation of the 21 cm Radio Line from the Early Universe. *Research in Astronomy and Astrophysics* **2020**, *20(7)*, 109.

31. Oks, E. Two Flavors of Hydrogen Atoms: a Possible Explanation of Dark Matter. *Atoms* **2020**, *8*, 33.

32. Gryzinski, M. Classical Theory of Atomic Collisions. I. Theory of Inelastic Collisions. *Phys. Rev.* **1965**, *138*, A336.

33. Fock, V. Zur Theorie des Wasserstoffatoms. *Z. Physik* **1935**, *98*, 145.

34. Callaway, J.; McDowell, M.R.C. What We do and do not Know About Electron Impact Excitation of Atomic Hydrogen. *Comments At. Mol. Phys.* **1983**, *13*, 19.

35. Whelan, C.T.; McDowell, M.R.C.; Edmunds, P.W. Electron Impact Excitation of Atomic Hydrogen. *J. Phys. B: At. Mol. Phys.* **1987**, *20*, 1587-1598.

36. Oks, E. Experiments on the Electron Impact Excitation of the 2s and 2p States of Hydrogen Atoms Confirm the Presence of their Second Flavor as the Candidate for Dark Matter, *Foundations* **2022**, *2*, 541-546.

37. Zammit, M.C.; Savage, J.S.; Fursa, D.V.; Bray, I. Electron-Impact Excitation of Molecular Hydrogen. *Phys. Rev. A* **2017**, *95*, 022708.

38. Wrkich, J.; Mathews, D.; Kanik, I.; Trajmar, S.; Khakoo, M.A. Differential Cross-Sections for the Electron Impact Excitation of the B $^1\Sigma_u^+$, c $^3\Pi_u$, a $^3\Sigma^+$, C $^1\Pi_u$, E, F $^1\Sigma^+$ and e $^3\Sigma_u^+$ States of Molecular Hydrogen. *J. Phys. B: At. Mol. Opt. Phys.* **2002**, *35*, 4695-4709.

39. Mason, N.J.; Newell, W.R. The Total Excitation Cross Section of the c $^3\Pi_u$ State of H_2. *J. Phys. B: At. Mol. Opt. Phys.* **1986**, *19*, L587-L591.

40. Oks, E. Experiments on the Electron Impact Excitation of Hydrogen Molecules Indicate the Presence of the Second Flavor of Hydrogen Atoms. *Foundations* **2022**, *2*, 697-703.

41. Fite, W.L.; Smith, A.C.H.; Stebbings, R.F. Charge Transfer in Collisions Involving Symmetric and Asymmetric Resonance. *Proceedings of the Royal Society,* **1962**, *A268*, 527-536.

42. Dalgarno, A.; Yadav, H.N. Electron Capture II: Resonance Capture from Hydrogen Atoms by Slow Protons. *Proc. Phys. Soc. (London)* **1953**, *A66*, 173.

43. Oks, E. Analysis of Experimental Cross-Sections of Charge Exchange between Hydrogen Atoms and Protons Yields another Evidence of the Existence of the Second Flavor of Hydrogen Atoms. *Foundations* **2021**, *1*, 265-270.

44. Bowman, J.D.; Rogers, A.E.E.; Monsalve, R.A.; Mozdzen, T.J.; Mahesh, N. An Absorption Profile Centred at 78 Megahertz in the Sky-Averaged Spectrum. *Nature* **2018**, *555*, 67-70.

45. Barkana, R. Possible Interaction between Baryons and Dark-Matter Particles Revealed by the First Stars. *Nature* **2018**, *555*, 71-74.

46. Jeffrey, N; Gatti, M; Chang, C; et al. Dark Energy Survey Year 3 results: Curved-sky weak lensing mass map reconstruction. *Monthly Notices of the Royal Astronomical Society* **2021**, *505(3)*, 4626–4645.

47. Oks, E. DES Map Shows a Smoother Distribution of Matter than Expected: A Possible Explanation. *Research in Astronomy and Astrophysics* **2021**, *21(10)*, 241–245.

48. Oks, E. Review of Latest Advances on Dark Matter from the Viewpoint of the Occam Razor Principle. *New Astronomy Reviews* **2023**, *96*, 101673.