## Wheatstone Bridge:

## Study of Resistance and Resistivity

Object: To measure resistance by means of the Slide Wire Wheatstone Bridge; to study the resistance of a wire as a function of length and cross sectional area; and to determine the resistivity of two different types of wire.

Apparatus: Slide-wire bridge, two unknown resistances, decade resistance box, dry cell, portable galvanometer, SPST momentary switch (normally open), sliding contact switch, board containing wires for resistivity measurements.

## FOREWORD

The resistance of a conductor is given by Ohm's law as the ratio of the potential difference V between its terminals to the current $I$ in the conductor, i.e., $R=\frac{V}{I}$. $R$ is given in ohms if $V$ is measured in volts and $I$ in amperes.

Many methods have been devised for measuring the resistance of a conductor. The simplest procedure makes use of Ohm's law as suggested above. An ammeter is used to measure the current supplied to a conductor by some voltage source, and a voltmeter is employed to determine the potential difference across the conductor. The resistance is now easily computed by using Ohm's law. While this method is extremely simple and was employed in Experiment 2EM, it is not widely used for the following reasons: (1) to measure a wide variety of resistances accurately would require meters having several ranges, (2) in computing the resistance $R$, the resistance of one meter must be known and taken into account, (3) the current heats the conductor and its resistance depends on its temperature; therefore, the measured resistance will depend on the magnitude of the current used, and (4) to get accurate measurements would require use of costly meters. To obviate these difficulties an apparatus known as a Wheatstone Bridge is employed which is considerably more versatile, accurate, and inexpensive.

A laboratory version of the Wheatstone Bridge is shown schematically in Figure 1. A resistance wire AB of uniform cross section, usually one meter in length, is mounted on a board with auxiliary components necessary for assembling the bridge. A variable decade resistance box R is connected between points A and C , and the unknown resistance X is connected between points C and B . Current to the bridge is furnished by a battery connected in series with the SPST switch S across AB. A galvanometer G is connected between C and a sliding contact switch $K$. The switch $K$ can be moved along the wire $A B$, thereby varying the length of the lower arms of the bridge, AK and KB .


Figure 1

Upon closing the switch $S$ and depressing the contact switch $K$, current generally exists in all arms of the bridge, including the galvanometer arm CK. However, by changing the position of the switch K along the wire AB , a point may be found at which the current through the galvanometer arm is zero. Under such a condition the galvanometer obviously indicates zero current and the bridge is considered to be balanced.

By the application of Ohm's law to the arms of the balanced bridge a simple and valuable equation can be obtained for the determination of X. Let the resistance of the wire between $A$ and $K$ be represented by $R_{A K}$ and that between $K$ and $B$ by $R_{K B}$. At the balance condition points $C$ and $K$ are at the same potential. Therefore, the potential difference from $A$ to $C$ must equal that from $A$ to $K$. If $I_{1}$ is the current in $R$ and $I_{2}$ the current in $R_{A K}$, it follows from Ohm's law that,
(1)

$$
\mathbf{I}_{1} \mathbf{R}=\mathbf{I}_{2} \mathbf{R}_{\mathrm{AK}} .
$$

Similarly,

$$
\begin{equation*}
\mathbf{I}_{1} \mathbf{X}=\mathbf{I}_{2} \mathbf{R}_{\mathrm{KB}} . \tag{2}
\end{equation*}
$$

By dividing equation (1) by equation (2) a unique relation is obtained involving the values of the resistances only, namely

$$
\frac{\mathbf{R}}{\mathbf{X}}=\frac{\mathbf{R}_{\mathrm{AK}}}{\mathbf{R}_{\mathrm{KB}}}
$$

(3) or

$$
\mathbf{X}=\frac{\mathbf{R}_{\mathrm{KB}}}{\mathbf{R}_{\mathrm{AK}}} \mathbf{R} .
$$

Since the wire AB is uniform in cross section, the resistance per cm k is assumed constant. If $\mathrm{L}_{1}$ denotes the length of $A K$ and $L_{2}$ that of $K B$, then $R_{A K}=k L_{1}$ and $R_{K B}=k L_{2}$. Substitution of these values into equation (3) yields,

$$
\begin{equation*}
\mathbf{X}=\frac{\mathbf{L}_{2}}{\mathbf{L}_{1}} \mathbf{R} \tag{4}
\end{equation*}
$$

If the lengths $L_{1}$ and $L_{2}$ can be measured accurately and the value of $R$ is known, $X$ can be determined easily from equation (4).

Analysis shows that the bridge is most accurate when X is equal to R . Therefore, in using a Wheatstone Bridge $R$ should be chosen so that a balance is achieved near the center of the wire $A B$, i.e., for the usual bridge near the 50 cm mark.

In a balanced condition the battery and the galvanometer may be exchanged without disturbing the balancing equation (4). However, a thorough analytical study of the balance condition shows that the bridge is most sensitive when the galvanometer is connected to the junction of the high resistance arms; in Figure $\mathbf{1}$ these arms are X and R and the bridge is shown set up in its most sensitive condition. The sensitivity of the bridge also increases with an increase in current in the arms, but the current used here is limited by the emf and internal resistance of the battery and the current-carrying capacity of the components in the arms.

Generally, the range of resistances measurable with the bridge vary from a few tenths of an ohm to several megohms. At lower values of X the resistances of the connecting wires can no longer be neglected; at higher values, the small current flowing through the galvanometer in an unbalanced condition makes it difficult to determine the null or balance point accurately.

The resistance of a wire varies directly as its length and inversely as its cross sectional area. This may be stated algebraically as,

$$
\begin{equation*}
\mathbf{R}=\frac{\rho \mathbf{L}}{\mathbf{A}}, \tag{5}
\end{equation*}
$$

where L is the length of a wire, A its cross sectional area, and $\rho$ the constant of proportionality which depends on the nature of the metal and its temperature. The above equation when solved for $\rho$, which is called the resistivity of the metal, gives,

$$
\begin{equation*}
\rho=\frac{\mathbf{R A}}{\mathbf{L}} \tag{6}
\end{equation*}
$$

If R is in ohms, A in $\mathrm{m}^{2}$, and L in meters, $\rho$ has units in the MKS system of ohm-meters.

## PROCEDURE

## Part I. Measurements of resistance with the slide wire bridge.

Compare Figure 1 and the laboratory slide wire bridge and note the location of points A, B, and C on the latter. Proceed to wire in the resistance box R and one unknown resistance X . Use the heaviest leads available in making these connections and make all connections secure. Most of the troubles arising in this experiment can be avoided by having tight connections between components. Connect the galvanometer to C and K using a long flexible lead for the latter connection. Now wire in the switch $S$ but do not close it until your circuit is checked by the instructor. The bridge is now ready for making the necessary measurements.

The proper way to obtain a balance point on the bridge is as follows: Initially set the resistance box R at a value between 10 and 30 ohms. After closing switch S, depress the switch K momentarily and note the deflection of the galvanometer. Then shift the switch K about 10 cm and again depress momentarily and note the deflection. These observations will probably provide sufficient information to determine whether the balance point is nearer or farther away than the original point of observation. By making several moves the balance point can be located accurately. Do not keep the switch K closed while sliding it along the wire, but rather make a move, depress switch K momentarily, observe the deflection, and then move along the meter stick to a new position for a new observation. Note that this requires the switch K to be closed for only a brief period of time. Such procedure will aid in finding the balance point quite accurately. If a balance point is not found, check the wiring for loose connections. If this fails to locate the difficulty secure aid from the instructor.

After having obtained a balance on the bridge note the position of this null point. If it is more than 10 cm from the center of the wire adjust $R$ until the balance point lies between 40 and 60 cm . Now each partner make two separate determinations for the null point and record the distances $L_{1}$ and $L_{2}$ to three significant figures. Please leave switch $S$ open except when making measurements. Record the value of R which is correct within $0.25 \%$. Now compute the value of X using the average $\mathrm{L}_{1}$ and $\mathrm{L}_{2}$.

In a like manner take similar data for the second resistor, the two resistors in parallel, and the two in series and perform all of the calculations indicated on the data and calculation summary sheet.

## Part II. Study of resistance of a wire as a function of length and cross sectional area. Determination of the resistivity of copper and nichrome.

On one of the resistance boards find spools with various lengths of the same size and type of (nichrome) wire and on the other resistance board find spools with various cross sections of the same length and type of (copper) wire. Take the data using the Wheatstone Bridge and then perform the calculations indicated on the data and calculation summary sheet. From equation (5) it appears that $\mathrm{R} \alpha \mathrm{L}$ and $\mathrm{R} \alpha 1 / \mathrm{A}$. To verify equation (5) make a plot of R vs L for the nichrome wire and a plot of R vs $1 / \mathrm{A}$ for the copper wire. From these plots determine the resistivity of the respective type of wire and compute the \% difference from the accepted handbook value.

## Questions and Problems

1. Prove that the battery and galvanometer can be interchanged without disturbing the balance condition.
2. If $R$ and $X$ were interchanged for resistor $a$, calculate the new balance point on the slide wire bridge.
DATA AND CALCULATION SUMMARY


| Physical Quantity being Calculated | Equation Used | Values Inserted | Answer |
| :--- | :--- | :--- | :--- |
| Slide wire value for $\mathrm{R}_{1}$ |  |  |  |
| Slide wire value for $\mathrm{R}_{2}$ |  |  |  |
| Slide wire value for $\mathrm{R}_{1}$ and $\mathrm{R}_{2}$ in series |  |  |  |
| Analytical value for $\mathrm{R}_{1}$ and $\mathrm{R}_{2}$ in series |  |  |  |
| $\%$ Diff. between slide wire and analytical value <br> for $\mathrm{R}_{1}$ and $\mathrm{R}_{2}$ in series |  |  |  |
| Slide wire value for $\mathrm{R}_{1}$ and $\mathrm{R}_{2}$ in parallel |  |  |  |
| Analytical value for $\mathrm{R}_{1}$ and $\mathrm{R}_{2}$ in parallel |  |  |  |
| \% Diff. between slide wire and analytical <br> value for $\mathrm{R}_{1}$ and $\mathrm{R}_{2}$ in parallel |  |  |  |

## Part III. Study of the resistance of a wire as a function of length and cross-sectional area.

 Determination of the resistivity of copper and nichrome.| Study of resistance as a function of length for Nichrome wire. |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Spool | Length <br> of wire | $\mathrm{L}_{1}$ | $\mathrm{~L}_{2}$ | R | $\mathrm{X}=\frac{\mathrm{L}_{2}}{\mathrm{~L}_{1}} \mathrm{R}$ | Diameter <br> of wire | Cross-Sect. <br> Area of wire |
| 1 |  |  |  |  |  |  |  |
| 2 |  |  |  |  |  |  |  |
| 3 |  |  |  |  |  |  |  |
| 4 |  |  |  |  |  |  |  |
| 5 |  |  |  |  |  |  |  |


| Study of resistance as a function of cross-sectional area for Copper wire. |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Spool | Length <br> of wire | $\mathrm{L}_{1}$ | $\mathrm{~L}_{2}$ | R | $\mathrm{X}=\frac{\mathrm{L}_{2}}{\mathrm{~L}_{1}} \mathrm{R}$ | Diameter <br> of wire | Cross-Sect. <br> Area of wire |
| 1 |  |  |  |  |  |  |  |
| 2 |  |  |  |  |  |  |  |
| 3 |  |  |  |  |  |  |  |
| 4 |  |  |  |  |  |  |  |
| 5 | $\nabla$ |  |  |  |  |  |  |


| Determination of Resistivity |  |
| :--- | :--- |
| Physical Quantity |  |
| Slope of the X vs L plot |  |
| Cross-sectional area of the nichrome wire |  |
| Experimental value for the resistivity of nichrome |  |
| Accepted value for the resistivity of nichrome |  |
| \% Difference between experimental and accepted values |  |
| Slope of the X vs $\frac{1}{\mathrm{~A}}$ plot |  |
| Length of the copper wire |  |
| Experimental value for the resistivity of copper |  |
| Accepted value for the resistivity of copper |  |
| \% Difference between experimental and accepted values |  |

