Convex and Discrete Geometry Prelim, June 2008
(Work only on eight of the questions)

1. Prove using Jensen inequality, that among all n-gons inscribed in a circle the regular one has the largest area.
2. State and prove Helly's theorem.
3. State and prove the area formula for spherical triangles.
4. Find the largest density which one can get by arranging nonoverlapping congruent circular discs in the plane.
5. Find the largest density which one can get by arranging nonoverlapping translates of a given triangle.
6. Show by example that the constant $d + 1$ in Caratheodory's theorem cannot bereplaced by d.
7. Let S be a planar compact convex set. Prove that there is a point in the plane such that $(-\frac{1}{2})S + u \subset S$. (Hint use Helly's theorem)
8. Let ABC be a spherical triangle. Let A' be the midpoint of BC, B' be the midpoint AC. Show that $A'B'$ is longer than half of AB.
9. Consider an arc c of the circle $x^2+y^2 = 1$ which lies in the first quadrant. Let a_x be the area of the region under the arc c and above the x-axis. Let a_y be the area to the left of the arc c and tot he right of the y-axis. Show that $a_x + a_y$ depends only on the length of the arc c.
10. Find the least number A such that for any two squares of combined area 1, a rectangle of area A exists such that the two squares can be packed in the rectangle (without interior overlap). You may assume that the sides of the squares are parallel to the sides of the rectangle.
11. Let C_1 and C_2 be circles whose centers are 10 units apart, and whose radii are 1 and 3. Find, with proof, the locus of all points M for which there exists points X on C_1 and Y on C_2 such that M is the midpoint of the line segment XY.