TOPOLOGY PRELIMINARY EXAM

5/27/00

Solve 8 out of the following problems, including **at least two of problems 9 through 12**. Use a separate sheet for each problem. Use a cover sheet which lists the problems you have chosen to solve.

Problem 1. Show that any separable metric space has a countable basis.

Problem 2. Prove that any product of completely regular spaces is completely regular.

Problem 3. Let $q : X \to Y$ be a quotient map, and $f : Y \to Z$ an arbitrary map from Y to a space Z. Show that f is continuous if and only if $f \circ q$ is continuous.

Problem 4. Let X be a space. Define the product $\alpha * \beta$ of paths in X, and show that if α is path-homotopic to α' and β is path-homotopic to β', then $\alpha * \beta$ is path-homotopic to $\alpha' * \beta'$.

Problem 5. Show that every completely metrizable space is a Baire space.

Problem 6. Prove that the collection of components of a space X forms a partition of X into closed subsets.

Problem 7. Let $X_0, X_1, \ldots$ be metrizable spaces. Prove that $\Pi_{n\in\mathbb{N}} X_n$ is metrizable.

Problem 8. Prove that if X and Y are connected, then $X \times Y$ is connected.

Problem 9. Let (X, d) be a compact metric space, and let $\mathcal{U}$ an open cover of X. Show that there exists $\epsilon > 0$ such that any subset of X of diameter less than ϵ is contained in some member of $\mathcal{U}$.

Problem 10. Prove that every compact Hausdorff space is normal.

Problem 11. Prove that regular Lindelöf spaces are paracompact. (A space is *Lindelöf* if every open cover has a countable subcover.)

Problem 12. Let b_0 be the point $(1, 0)$ on the unit circle S^1. In the proof that $\pi_1(S^1, b_0)$ is isomorphic to the group $\mathbb{Z}$ of integers under addition, describe how the isomorphism $\phi : \pi_1(S^1, b_0) \to Z$ is defined. (You need not prove it is an isomorphism, only define it.)