TENTATIVE LIST OF 620-21 TOPICS (AND SOME 520-21-22 TOPICS) TO BE REVIEWED FOR 1992 WRITTEN GENERAL EXAM IN ANALYSIS:

PRIMARY TOPICS FOR WHICH YOU SHOULD KNOW DEFINITION, EASIER SELF-CONTAINED PROOFS, EXAMPLES:

1. Continuity and differentiability of real functions. Sets of continuity. Continuous nowhere differentiable functions.
2. Monotone functions and functions of bounded variation.
3. Cardinality, countable sets, uncountable sets, sets of cardinality c.
4. Riemann integral, characterization (bounded, discountinuity set of measure zero) of Riemann integrability of a function on an interval.
5. Lebesgue outer measure λ° and Lebesgue measure λ on an interval and on $\mathbb{R}$. Non-measurable sets.
6. Perfect sets and Cantor sets (of measure zero and of positive measure).
7. Nowhere dense sets, first and second category sets, residual sets, the Baire Category Theorem, first category sets of full measure.
8. σ-algebras, mininal σ-algebra containing a given collection of sets, the Borel sets. Finite countably-additive non-negative measures μ on a σ-algebra.
9. Lebesgue Differentiability Theorem (not the proof).
10. Measurable functions, simple functions, (Lusin's Theorem)
11. Sequences of measurable functions. Uniform, pointwise, almost everywhere, and L^p-convergence (primarily L^1-, L^2-, and L^∞-convergence). Convergence in measure. Egoroff's Theorem. Non-negative measureable functions are a.e. limits of monotone increasing sequences of non-negative simple functions.
12. Legesgue integral (with respect to Lebesgue measure λ and with respect to a finite non-negative measure μ). Dominated Convergence Theorem and Monotone Convergence Theorem.
13. Absolutely continuous functions ($\varepsilon - \delta$ partition definition and equivalent Lebesgue integral definition). Absolute continuity of one measure with respect to another and the Radon-Nikodym Theorem (Royden pg. 238-9, just the facts, not the proof.)

14. Complete normed linear spaces, in particular L^p spaces (primarily L^1, L^2, and L^∞), with respect to Lebesgue measure λ on [0,1] and $\mathbb{R}$ and with respect to an arbitrary finite non-negative measure μ. Hölder's inequality in L^p and Schwarz's inequality in L^2. Completeness of L^p (primarily for p=1,2, and ∞ (STATEMENTS). Also ℓ^p spaces.

ADDITIONAL TOPICS FOR WHICH YOU SHOULD KNOW THE DEFINITIONS AND THE MAIN BASIC THEOREMS (not the proofs):

15. Outer measure and associated measure (Royden Sec. 12. 1).
16. Caratheodory Theorem on extension to σ-algebra of measure defined on an algebra (Royden Sec. 12.2).
17. Product measures and Fubini's Theorem (Royden Sec. 12.4). STATEMENT.