Weighted Approximation with the Bernstein-Chlodovsky Operators

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Abstract

In 1937, I. Chlodovski introduced a modification of the Bernstein polynomial operators which could be used for approximation of functions $f \in C[0, \infty)$ and produced several interesting results. However, his results did not address weighted approximation, in which, one introduces a weight function such as $W(x) = e^{-x^{\alpha}}$. One then considers the approximation of functions $f \in C_W[0, \infty)$, the space of continuous functions f for which $\|f\|_W = \sup_x |W(x)f(x)|$ is finite, and for which $W(x)f(x) \to 0$ as $x \to \infty$, by similarly weighted polynomials. Similar questions may be considered, of course, for functions in $C_W(-\infty, \infty)$, in which space $W(x)f(x) \to 0$ as $|x| \to 0$.

For years, any question of using the Bernstein-Chlodovski operators for weighted approximation was dismissed out of hand as imposslble. Here, some recent progress in using the "impossible" Bernstein-Chlodovski operators for weighted approximation will be presented, and some unsolved open problems will be described.