High-beta Extended MHD Simulations of Stellarators

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Abstract

It is desirable to understand the high beta properties of stellarator plasmas. To study these effects we use the nonlinear, extended MHD code NIMROD to model 3D magnetic topology evolution as beta is increased from vacuum. Such studies require the use of the newly recovered 3D semi-implicit operator to guarantee accurate temporal convergence. Spatial convergence studies have also been performed to ensure accurate final results. The configurations under investigation are an l=2, M=5 torsatron with geometry modeled after the Compact Toroidal Hybrid (CTH) experiment and an l=2, M=10 torsatron capable of having vacuum rotational transform profiles near unity. Finite beta plasmas are created using a volumetric heating source and temperature dependent anisotropic heat conduction and resistivity. The onset of MHD instabilities and nonlinear consequences are monitored as a function of beta as well as the fragility of the magnetic surfaces.

Overview and Background

- This study comprises the first high temperature test of the NIMROD code initialized from toroidal, non-axisymmetric plasma conditions
- For computational convenience the density in simulations is uniform and fixed

NIMROD

- Solves the following equations in an axisymmetric domain

$$\rho\frac{\partial \boldsymbol{v}}{\partial t} + \rho(\boldsymbol{v}\cdot\boldsymbol{\nabla})\boldsymbol{v} = \boldsymbol{J}\times\boldsymbol{B} - \boldsymbol{\nabla}p + \boldsymbol{\nabla}\cdot\nu\rho W,$$

$$W = \boldsymbol{\nabla}\boldsymbol{v} + (\boldsymbol{\nabla}\boldsymbol{v})^T - \frac{2}{3}\boldsymbol{\nabla}\cdot\boldsymbol{v}I,$$

$$\frac{n_0}{\gamma-1}\left(\frac{\partial T}{\partial t} + \boldsymbol{v}\cdot\boldsymbol{\nabla}T\right) = -\frac{p}{2}\boldsymbol{\nabla}\cdot\boldsymbol{v} - \boldsymbol{\nabla}\cdot\boldsymbol{q} + Q,$$

$$\boldsymbol{q} = -n\left[\chi_{\|}\hat{\boldsymbol{b}}\hat{\boldsymbol{b}} + \chi_{\perp}(I - \hat{\boldsymbol{b}}\hat{\boldsymbol{b}})\right]\cdot\boldsymbol{\nabla}T,$$

$$\chi_{\|} = \chi_{\|,0}\left(\frac{T}{T_0}\right)^{5/2},$$

$$\chi_{\perp} = \chi_{\perp,0}\left(\frac{T}{T_0}\right)^{-1/2}|\boldsymbol{B}|^{-2},$$

$$\frac{\partial \boldsymbol{B}}{\partial t} = -\boldsymbol{\nabla}\times\boldsymbol{E} + \kappa_{divB}\boldsymbol{\nabla}\boldsymbol{\nabla}\cdot\boldsymbol{B},$$

$$\boldsymbol{E} = -\boldsymbol{v}\times\boldsymbol{B} + \eta\boldsymbol{J},$$

$$\eta = \eta_0\left(\frac{T}{T_0}\right)^{-3/2},$$

$$\mu_0\boldsymbol{J} = \boldsymbol{\nabla}\times\boldsymbol{B},$$

Configuration Parameters

- Physical input parameters for l=2, M=5 torsatron cases are:

$$R_0 = 0.75\text{ m},\quad a = 0.3\text{ m},\quad B_{\phi 0} = 0.55\text{ T},\quad v_A = 3.1\times10^6\text{ m/s},\quad \eta_0 = 7.9\times10^{-7}\ \Omega\cdot\text{m},\quad S = 9.3\times10^5,\quad \nu = 0.63\text{ m}^2/\text{s},\quad P_m = 1,\quad n_0 = 1.5\times10^{19}\text{ m}^{-3},\quad T_0 = 290\text{ eV},\quad \frac{\chi_{\|,0}}{\chi_{\perp,0}} = 10^5,$$

- Physical input parameters for l=2, M=10 torsatron cases are:

$$R_0 = 3.9\text{ m},\quad a = 0.93\text{ m},\quad B_{\phi 0} = 0.425\text{ T},\quad v_A = 1.31\times10^6\text{ m/s},\quad \eta_0 = 8.2\times10^{-7}\ \Omega\cdot\text{m},\quad S = 7.1\times10^4,\quad \nu = 0.65\text{ m}^2/\text{s},\quad P_m = 1,\quad n_0 = 2.5\times10^{19}\text{ m}^{-3},\quad T_0 = 400\text{ eV},\quad \frac{\chi_{\|,0}}{\chi_{\perp,0}} = 10^5,$$

- $B_{\phi 0}$ is the vacuum magnetic field on axis

Heating Source

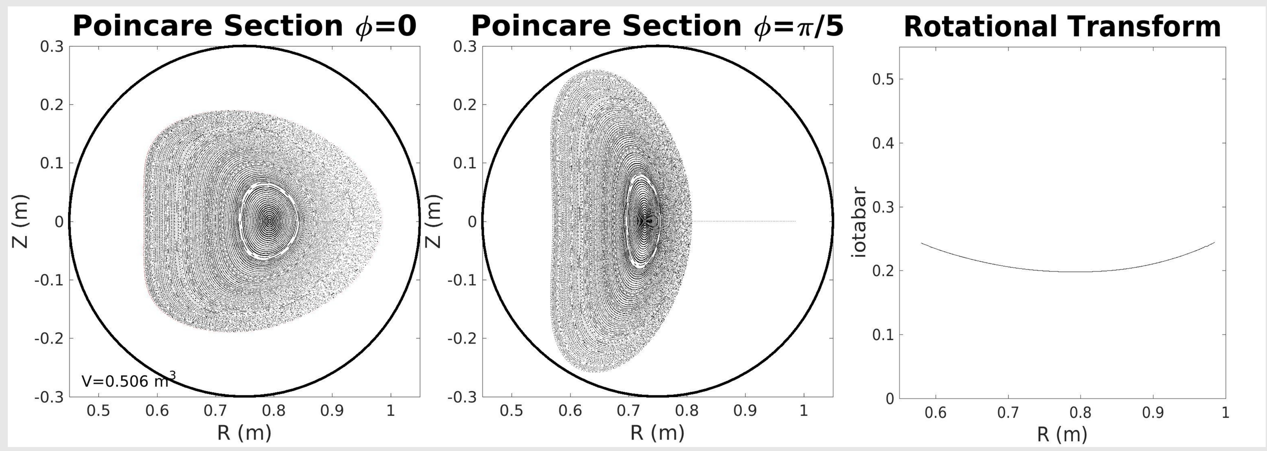
- In all the results shown heat is added by an ad hoc volumetric source
- The source is applied uniformly throughout the computational domain
- The source is ramped up at the start of the simulation following a hyperbolic tangent curve in time

Vacuum Configurations

- Simulations are initialized from NIMROD vacuum magnetic field configurations.

l=2, M=5 Torsatron

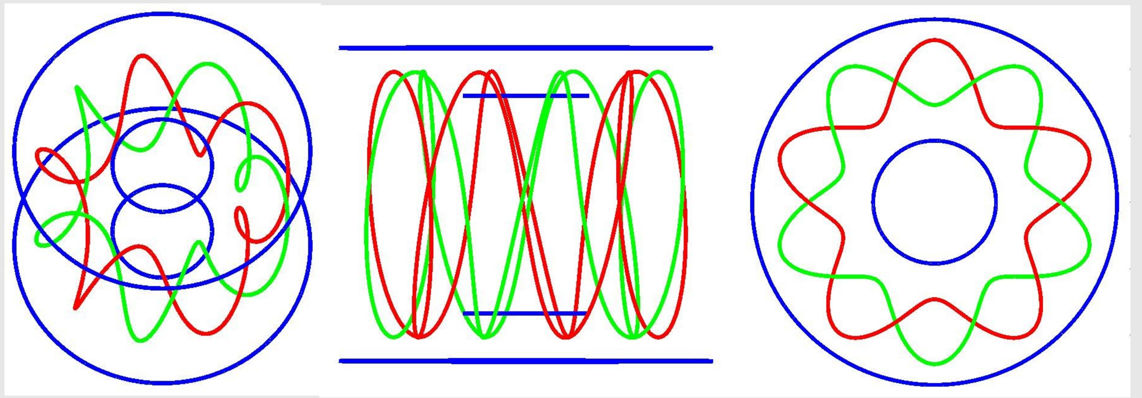
- Magnetic field at the domain boundary is loaded from reconstruction of the Auburn Compact Toroidal Hybrid (CTH)
- The rotational transform is artificially raised above CTH values by increasing radial boundary fields in the following manner:
  - Fields are Fourier decomposed as $B_r(\theta,\phi) = \sum_{m,n} B_{m,n}e^{im\theta - in\phi}$
  - Magnitudes of the dominant toroidal modes (n = 5) are increased
  - Outward flux surface shifts are corrected with vertical field (n = 0, m = 1)



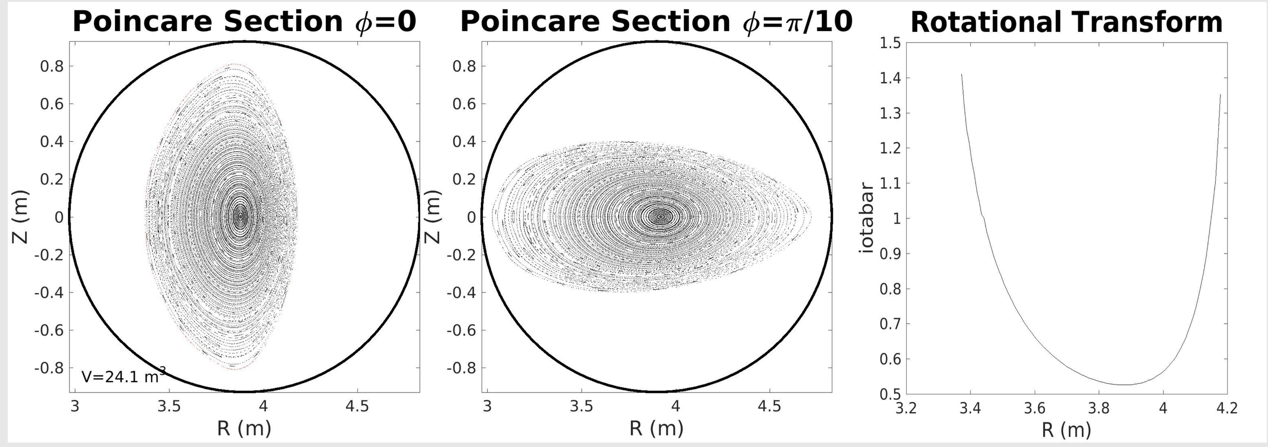
- Magnetic islands appear and plasma volume is lost as the rotational transform approaches one half

l=2, M=10 Torsatron

- Larger rotational transform profiles can be achieved in torsatrons with higher field period, M
- Configurations are created using the Integrable Field Torsatron (IFT) coil optimization code
- Helical coil pitch and vertical field dipole and quadrupole magnitude are optimized to reduce magnetic island sizes, center magnetic axis, and obtain a large rotational transform
- The helical winding law being used is: $\phi = \frac{\ell}{M}\theta - \alpha_1\sin(\theta) - \alpha_2\sin(2\theta) - \alpha_3\sin(3\theta)$
- Coils are treated as point currents (no cross sectional area or shape) with the following orientation



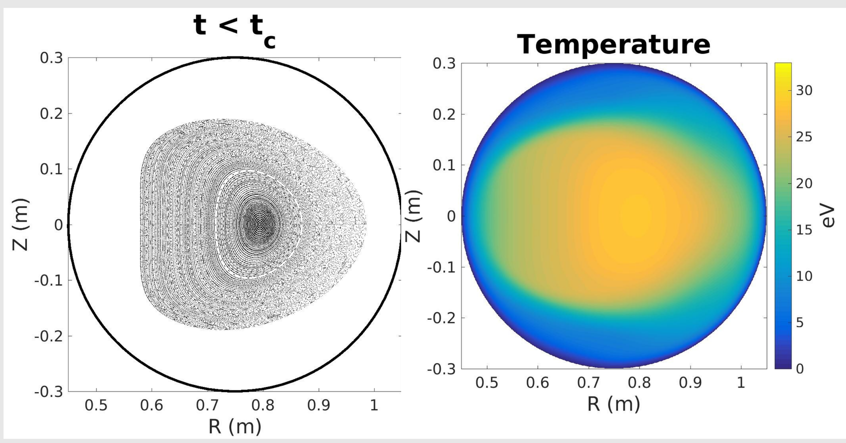
- The resulting magnetic configuration is



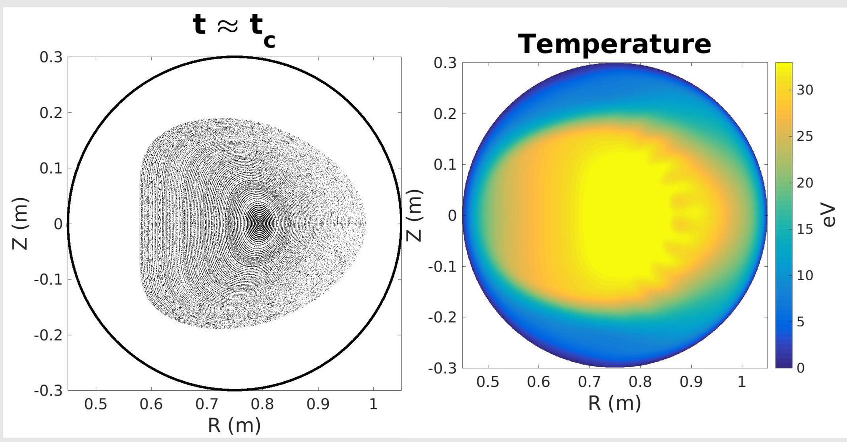
Heating Degrades Confinement for l=2, M=5

- The torsatron is heated volumetrically until thermal confinement is lost
- Simulations are simplified by keeping plasma resistivity and perpendicular thermal conduction fixed

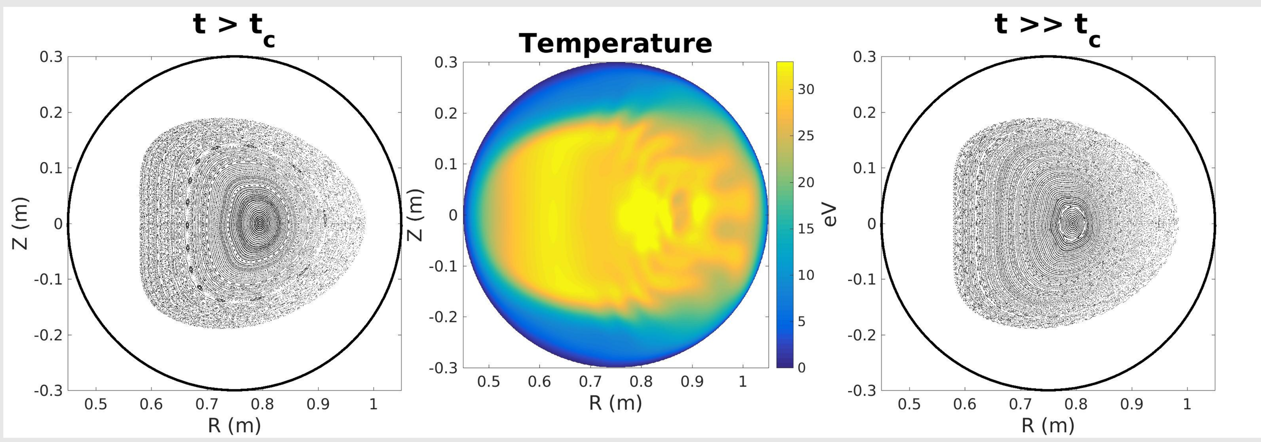
Evolution



- The temperature profile is initially peaked
- Once some threshold is passed at t = t_c small edge islands begin to form

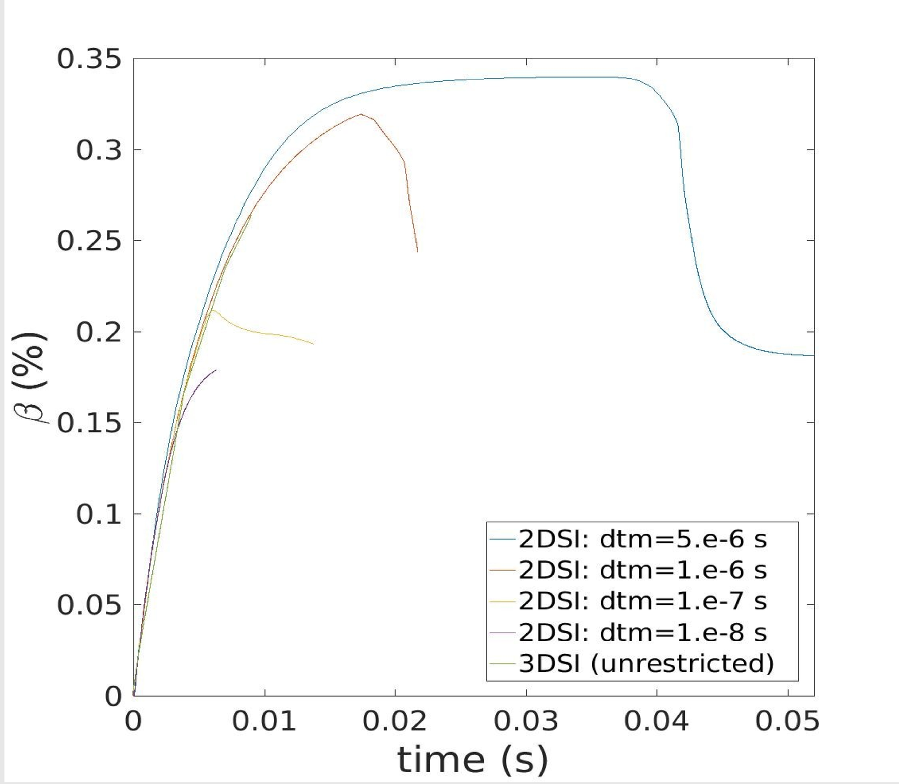


- Edge stochastization follows from island overlap
- Stochastization significantly degrades thermal confinement
- As the temperature of the plasma drops some of the islands heal, but others, including the n=1, m=5 near the core, remain



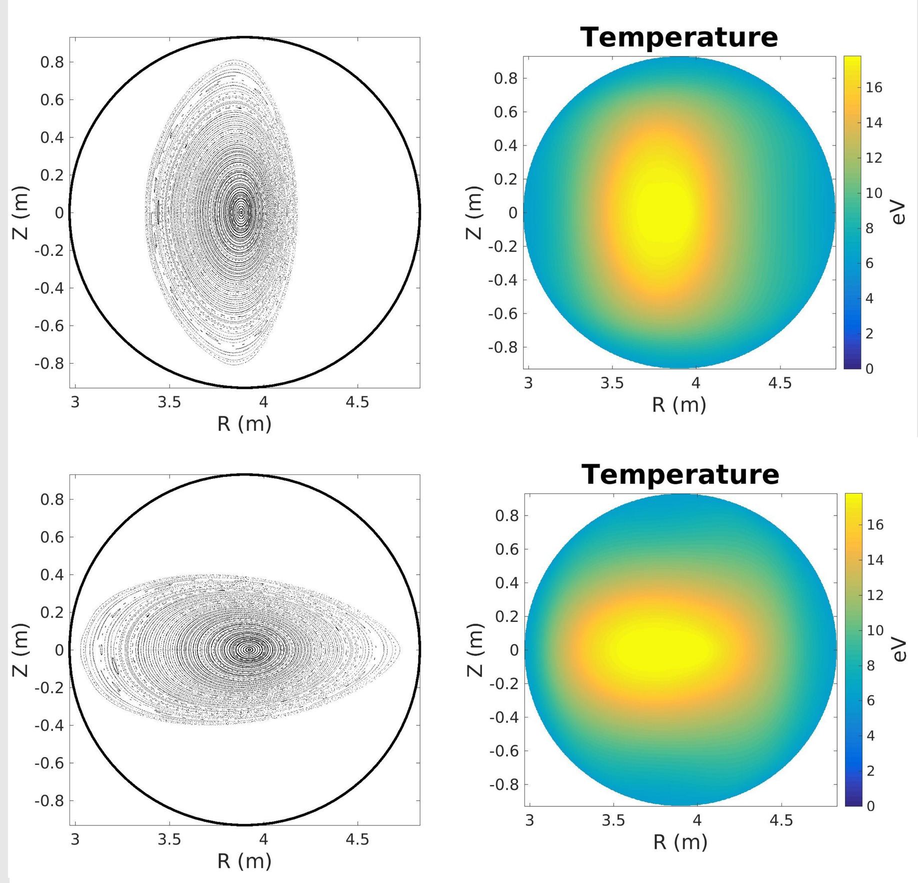
Critical Beta

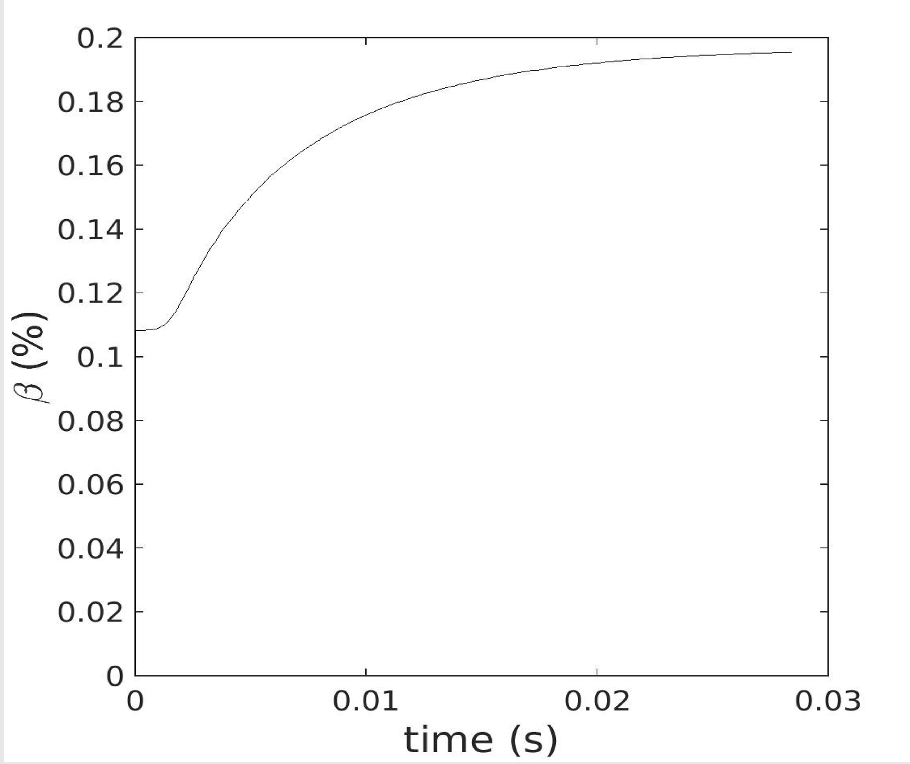
- The critical beta for confinement degradation is seen to depend sensitively on the temporal convergence of the simulations
- dtm is the maximum allowed time step
- The evolutions of each case have similar qualitative behavior but differ quantitatively
- The standard 2 dimensional semi-implicit operator (2DSI) is expected to have good convergence for 3D simulations at dtm=1.e-8 (see Roberds P1.0101)
- The new 3 dimensional semi-implicit operator (3DSI) appears to also require a restricted time step for accurate results (tests in progress)
- The thermal diffusion time of approximately 0.077 seconds is never reached in simulations



No Degradation Seen in Preliminary l=2, M=10

- The l=2, M=10 torsatron is expected to have better stability properties
- Low beta simulations of this device show no early signs of confinement loss
- The thermal diffusion time of 0.7 seconds has not yet been reached in simulations





Future Work

- Find time step restriction for accurate 3DSI temporal convergence of l=2, M=5 torsatron
- Test spatial convergence of l=2, M=5 torsatron
- Consider more computationally tractable simulation methods (equilibrium initialization, smaller geometries, simplified physics, etc.)
- Consider other configurations with different stability properties
- Introduce momentum sources to study flow effects

Summary

- Initial conditions are 3D magnetic flux surfaces
- High beta plasmas are generated with an ad hoc heating source
- Simulations show loss of confinement in l=2, M=5 torsatron
- Onset of confinement loss is dependent upon temporal convergence
- No confinement loss has been seen in simulations of l=2, M=10 torsatron