Simulations of Sawtooth Oscillations in CTH

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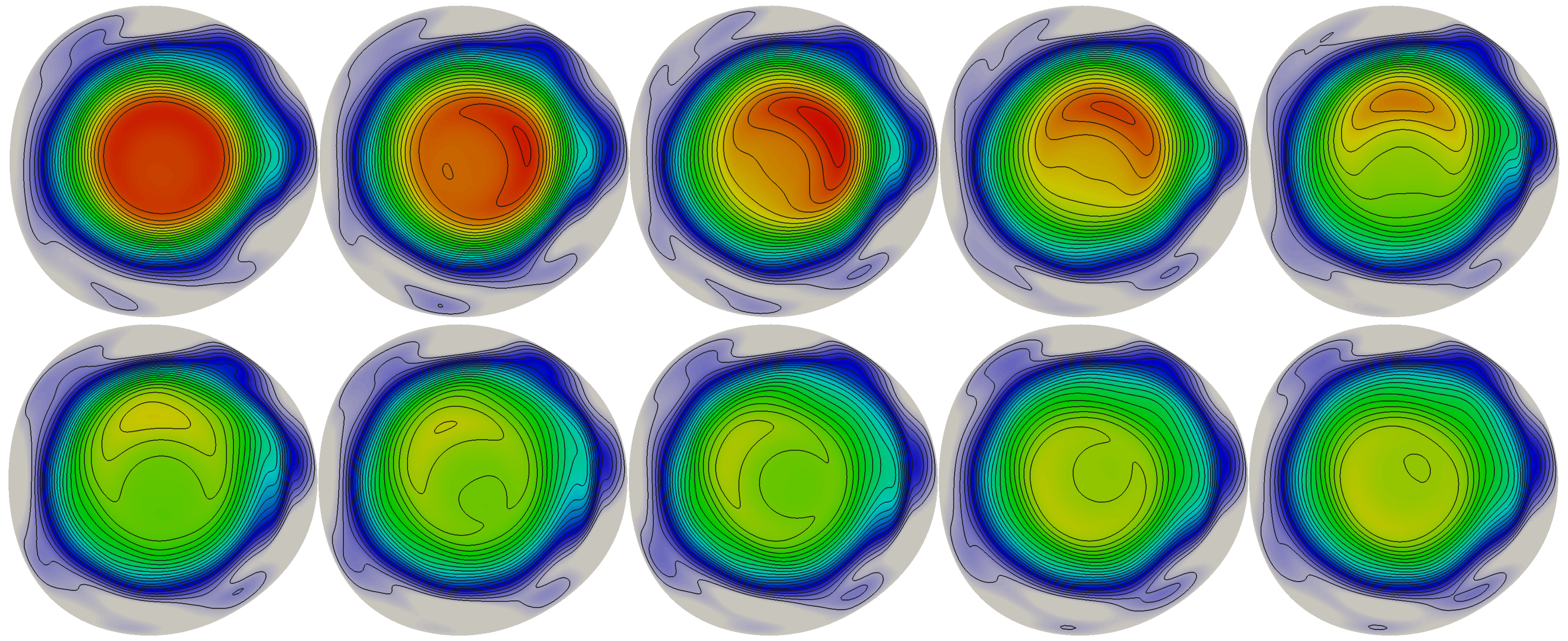
Introduction

Sawtoothing is a repeated relaxation that occurs in the cores of tokamak devices under many operating conditions. The relaxations flatten temperature and safety factor profiles, limiting the central safety factor to $q_0 \gtrsim 1$. Sawtoothing can play an important role in determining tokamak profile shapes and the regions of parameter space that permit disruption free operation. The relaxations eject thermal energy from the core into the outer region of the plasma, affecting energy and particle confinement. An early theoretical picture for sawtooth relaxations that had much success in describing the behavior of small ohmic tokamaks is the Kadomptsev model. This model describes a relaxation as the resistive reconnection of the plasma core driven by the evolution of an unstable $m, n = 1{,}1$ tearing mode.

Because CTH is a small ohmically heated experiment, we have choosen to study sawtoothing in CTH by evolving extended MHD equations with NIMROD. In the past, extended MHD simulations of small ohmic tokamaks have given reasonable values for τ_{saw} and τ_{crash}. Ideal MHD equilibria from the 3D equilibrium code VMEC are used for initial conditions.

SXR Tomography

- Implementation of Fourier-Bessel SXR tomography in python
- Basis functions can use flux from a VMEC equilibrium file as radial coordinate
  - Allows knowledge of magnetic field structure to inform interpretation of SXR data.
- Outputs reconstructions to .vtk format for flexible visualization
- Results sensitive to number of basis functions and SVD cutoff number
- Toroidal plasma rotation is evident from the poloidal rotation of the reconnecting core.



Acknowledgements

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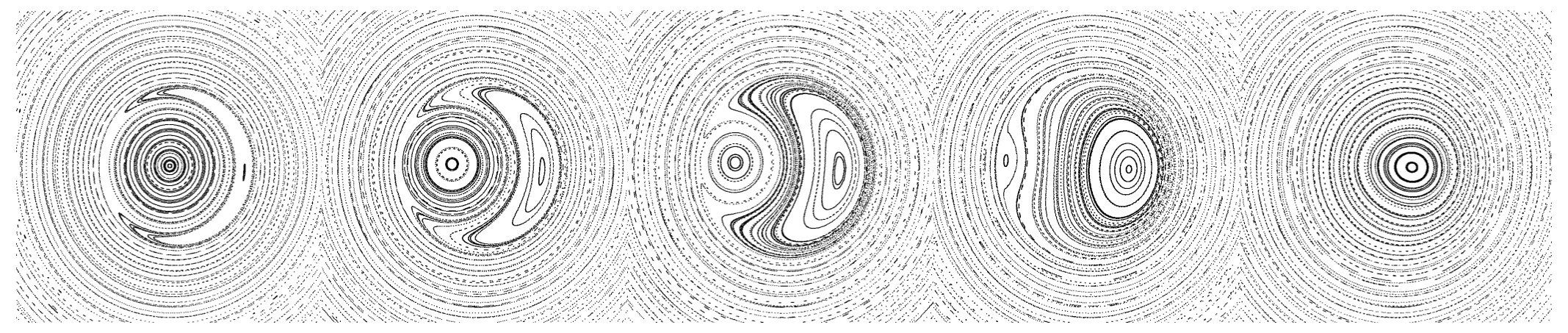
Sawtooth Simulations of CTH Running in Tokamak Mode

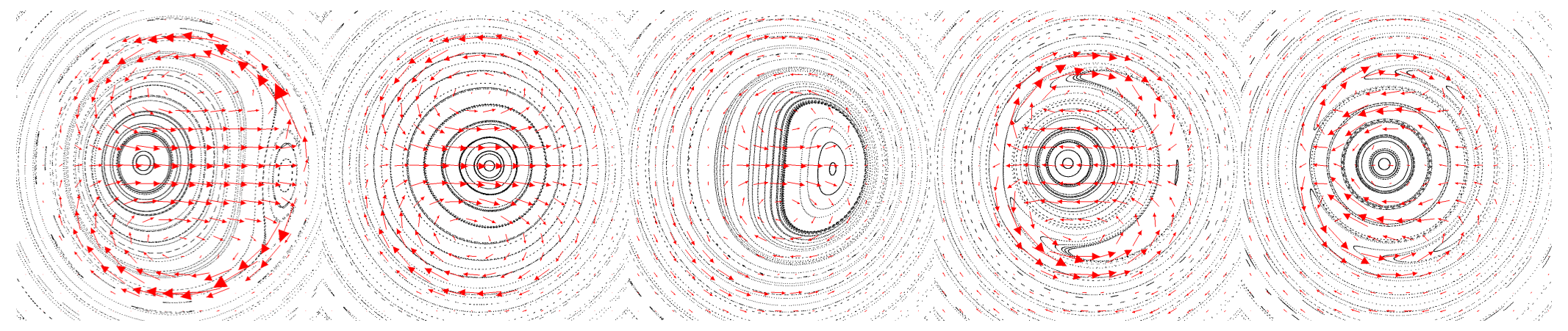
- Starting from initial conditions with $q_0 > 1$, the total plasma current is ramped to the desired amount

1. After $q_0 < 1$, the internal kink mode becomes unstable and is excited at a low amplitude.
2. The mode grows exponentially until it becomes large and enters the non-linear stage of evolution. Then an island grows rapidly, driving the reconnection of the plasma core.
   - The relaxation has the effect of flattening the temperature and current density profiles. The flattened profiles
3. Become peaked again as the core is reheated and the entire process starts over at step 1.



This is the picture of sawtoothing which was predicted by Kadomptsev and verified by many numerical simulations in the past.

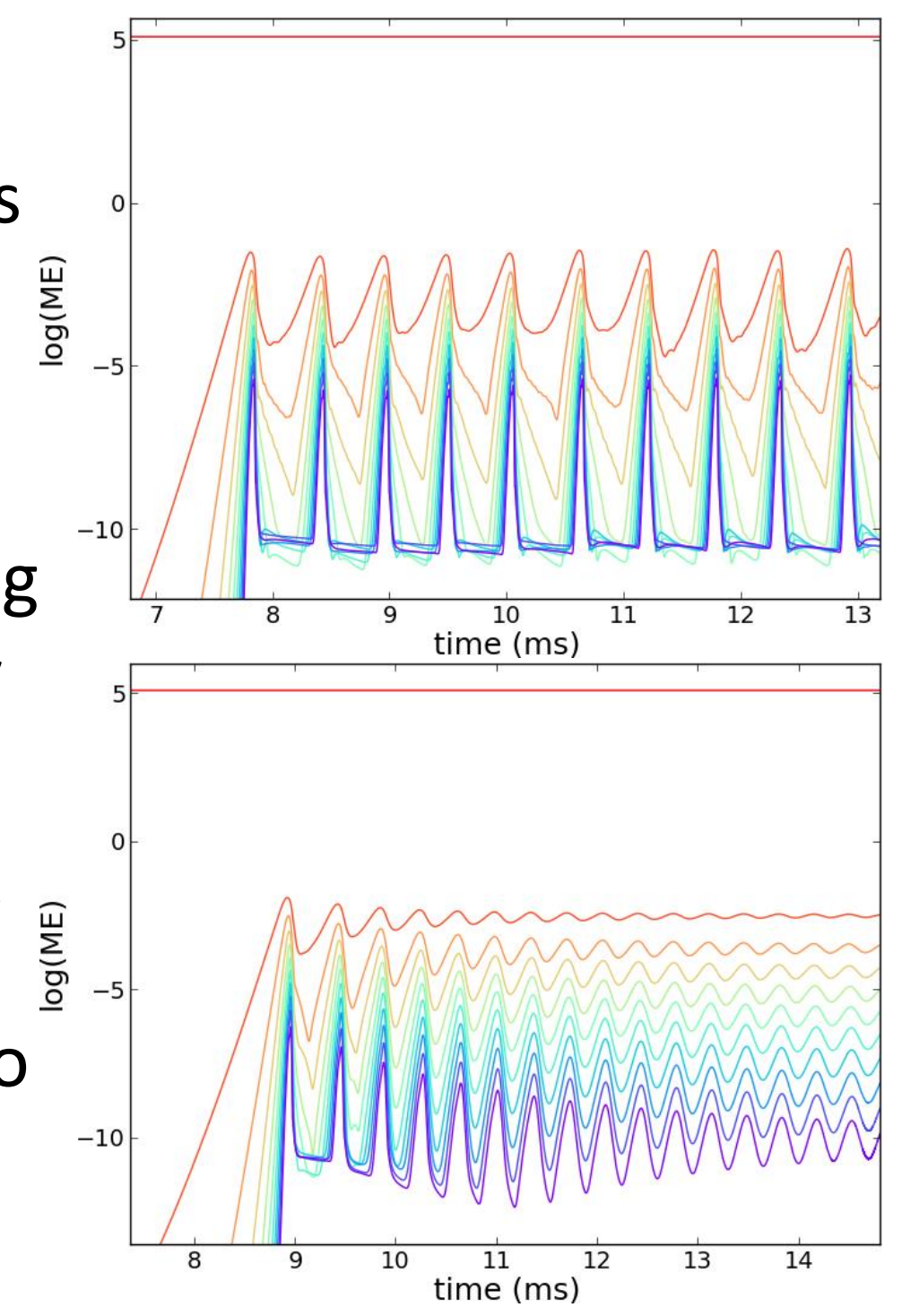
- Secondary MHD activity sometimes follows relaxations
  1. Tearing modes
     - May grow to large size and cause large amounts of stochasticity
     - Measures taken to prevent large tearing modes after crash
       - $q = 2$ surface near wall for wall stabilization
       - $T_{wall} = 30\ eV$ reduces growth rates or stabilizes tearing modes
  2. Flux re-arrangement due to strong return flow
     - Can be suppressed by
       - increased viscosity
       - reducing $k_\perp$ while holding S constant (causes faster reheating of core)



Some relaxations are followed by a flux-rearrangement immediately after the crash.

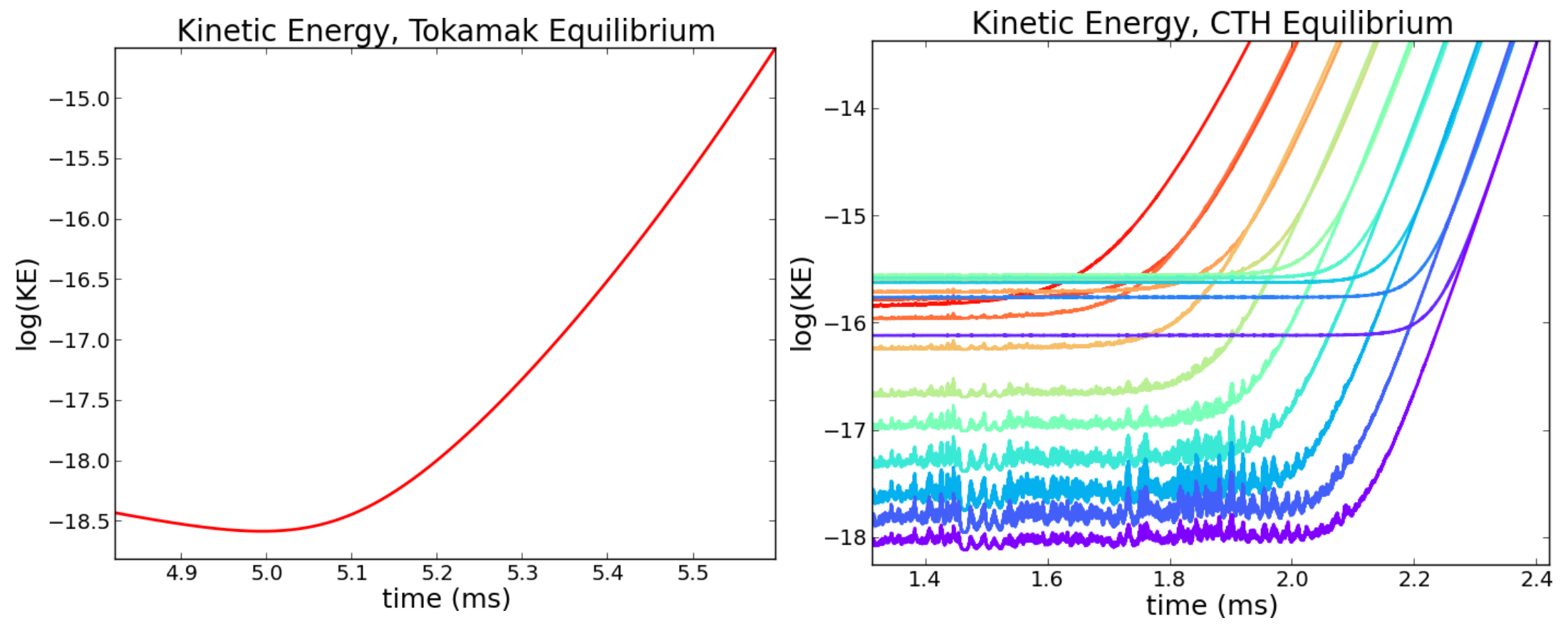
- Obtaining solutions that reach an oscillating steady state requires careful selection of dissipation, diffusion and source parameters, including $k_\perp, \mathrm{k}_\parallel, \nu, I_{plasma}$
- Minimum value of $k_\parallel$ needed
- Oscillating steady states tend to be found for small values of $k_\perp, \beta_p$.

An oscillating steady state is found, but a solution that relaxes to a non-oscillating helical steady state is found when I_{plasma} is reduced from 105kA to 102.5kA



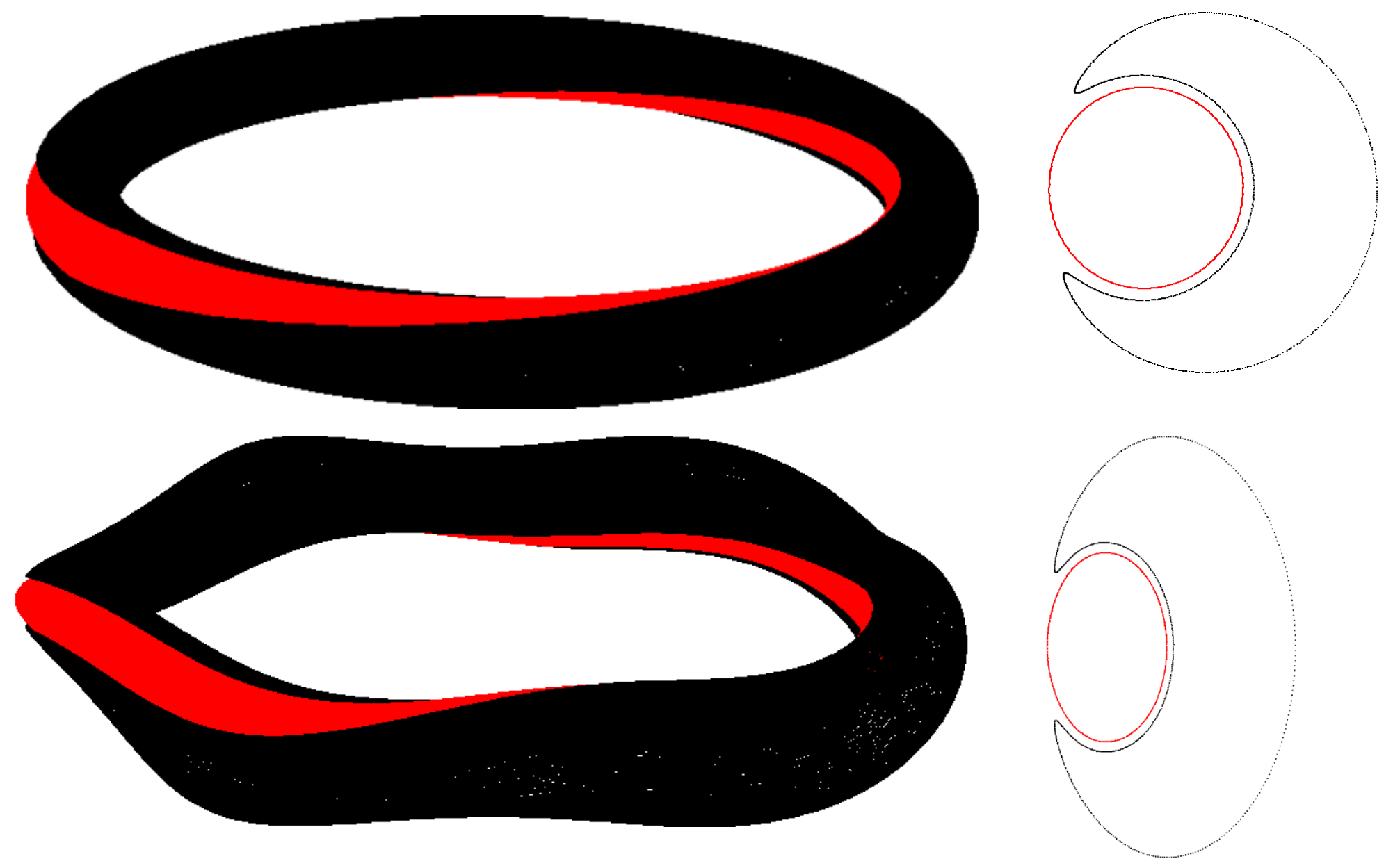
Sawtooth Simulations of CTH With Helical Stellarator Field

- After $q_0 < 1$, the internal kink mode becomes unstable and is excited at a low amplitude.
  - Unstable mode is represented with Fourier numbers $n = 1{,}4{,}6{,}9{,}11{,}14{,}16, \ldots$
  - Can be said that n is no longer a good quantum number when helical fields are added
  - The energies of these Fourier numbers grow at the same rate since they are representing the same unstable mode.



A $m, n = 1{,}1$ tearing mode is excited when q_0 drops below 1. In a tokamak, this mode is characterized by $n = 1$. As 3D stellarator fields are added, the mode requires more Fourier numbers to represent.

- In the nonlinear evolution, an island surface grows and the center of the island becomes the new magnetic axis after the plasma core is completely reconnected.
- Compared to the tokamak case, the island and core are both helically deformed.



The growing island (black) and reconnecting plasma core (red) are shown at a late stage of the relaxation process. In the case with stellarator fields turned on, the structure of the core and island are deformed.

- A correlation, in simulation and experiment, is observed between
  - Strength of stellarator field
  - Sawtooth period τ_{saw}
- As stellarator field is increased, confinement is degraded and temperature is lowered
  - Smaller generalized minor radius $\left(a = \frac{2V}{S}\right)$
  - Small chains of islands in equilibrium

As the 3D stellarator field is added, confinement is degraded and plasma temperature is reduced. This may explain faster sawtooth frequency as stellarator field is added.