WEAK LOG MAJORIZATION AND DETERMINANTAL INEQUALITIES

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ABSTRACT. Denote by $\mathbb{P}_n$ the set of $n \times n$ positive definite matrices. Let $D = D_1 \oplus \cdots \oplus D_k$, where $D_1 \in \mathbb{P}_{n_1}, \ldots, D_k \in \mathbb{P}_{n_k}$ with $n_1 + \cdots + n_k = n$. Partition $C \in \mathbb{P}_n$ according to $(n_1, \ldots, n_k)$ so that $\operatorname{Diag} C = C_1 \oplus \cdots \oplus C_k$. We prove the following weak log majorization result:

$$\lambda(C_1^{-1}D_1 \oplus \cdots \oplus C_k^{-1}D_k) \prec_{w \log} \lambda(C^{-1}D),$$

where $\lambda(A)$ denotes the vector of eigenvalues of $A \in \mathbb{C}_{n\times n}$. The inequality does not hold if one replaces the vectors of eigenvalues by the vectors of singular values, i.e.,

$$s(C_1^{-1}D_1 \oplus \cdots \oplus C_k^{-1}D_k) \prec_{w \log} s(C^{-1}D)$$

is not true. As an application, we provide a generalization of a determinantal inequality of Matic [2, Theorem 1.1]. In addition, we obtain a weak majorization result which is complementary to a determinantal inequality of Choi [1, Theorem 2] and give a weak log majorization open question.

REFERENCES

[1] D. Choi, Determinantal inequalities of positive definite matrices, Math. Inequal. Appl. **19** (2016), 167-172.

[2] I. Matic, Inequalities with determinants of perturbed positive matrices, Linear Algebra Appl. **449** (2014), 166-174.