The qpr-sequence

A *principal* minor of a matrix is the determinant of a (square) submatrix whose row and column indices are the same. The *enhanced principal rank characteristic sequence* (*epr-sequence*) of a symmetric matrix $B \in \mathbb{R}^{n\times n}$ is $\ell_1\ell_2\cdots\ell_n$, where ℓ_k is `A` (respectively, `N`) if all (respectively, none) of the principal minors of order k are nonzero; if some but not all are nonzero, then $\ell_k = \mathtt{S}$. Due to the numerous applications of principal minors, epr-sequences have received considerable attention since their introduction.

An *almost-principal* minor of a matrix is the determinant of a (square) submatrix whose row and column indices differ in exactly one index. Motivated by the fact that principal and almost-principal minors have applications in algebraic geometry, statistics, theoretical physics and matrix theory, for example, we have introduced a new sequence that extends the epr-sequence by also taking into consideration the almost-principal minors of the matrix. A minor of a matrix is *quasi-principal* if it is a principal or an almost-principal minor. The *quasi principal rank characteristic sequence* (*qpr-sequence*) of a symmetric matrix $B \in \mathbb{R}^{n\times n}$ is $q_1q_2\cdots q_n$, where q_k is `A` (respectively, `N`) if all of (respectively, none of) the quasi-principal minors of order k are nonzero; if some but not all are nonzero, then $q_k = \mathtt{S}$.

In this talk, a complete characterization of the qpr-sequences that are attainable by real, symmetric matrices will be presented. This characterization establishes a contrast between qpr- and epr-sequences, as the latter are still far from being characterized.