Matrix monotone functions, means, and the geometry of $\mathbb{P}_n$

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Abstract

Denote by $\mathbb{H}_n$ the set of all n-by-n Hermitian matrices, and $\mathbb{P}_n$ the set of all n-by-n positive definite matrices. For $A \in \mathbb{H}_n$. we write $A \geq 0$ if A is positive semidefinite. The Löwner partial order $\leq$ on $\mathbb{H}_n$ is given by $A \leq B$ if and only if $B - A \geq 0$. A function $f : (a, b) \to \mathbb{R}$ is said to be *matrix monotone* if $A \leq B$ implies $f(A) \leq f(B)$ for all $A, B \in \mathbb{P}_n$ with spectra contained in (a, b). We discuss major results in the theory of matrix monotone functions and its connection with special maps called *matrix means* (the Ando-Kubo Theorem). Finally, we look at the geometry of $\mathbb{P}_n$ as developed by Bhatia.