log-majorization inequalities involving geometric mean of matrices

Let $\mathbb{R}^+$ denote the set of all positive real numbers. For $a, b \in \mathbb{R}^+$, the geometric mean is $\sqrt{ab}$. In the extension for two matrices, a good platform is $\mathbb{P}_n$, the set of all $n \times n$ positive definite matrices. The challenge is that the product of two positive definite matrices is not necessarily positive definite and $\sqrt{AB}$, where $A, B \in \mathbb{P}_n$, is not an appropriate definition always. Operator theory and differential geometry are two views regarding the extension of the definition of geometric mean from $\mathbb{R}^+$ to $\mathbb{P}_n$. Since differential geometry point of view gives us a good understanding of the geometric mean of two positive definite matrices, after reviewing the Riemannian structure of $\mathbb{P}_n$, and the geometric mean in terms of geodesic, we will present some log-majorization inequalities and norm inequalities involving geometric mean. Furthermore, very recent inequalities of Lemos and Soares involving geometric mean will be presented.