Thinking outside the matrix

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In the late 60's, Ryser and Woodall independently came up with a bordering technique to change an $n \times n$ $(0,1)$ matrix A satisfying:

$$A^T A = D + \lambda J$$

with D a positive diagonal matrix, J the all 1's matrix and $\lambda > 0$ into an $(n+1) \times (n+1)$ (complex) matrix $\overline{A}$ satisfying:

$$\overline{A}^T \overline{A} = I.$$

I'll state and prove this, as it is a nice linear algebra theorem.

This was done in order to attack the λ-conjecture, still open 40 plus years hence.

Combinatorial double-counting techniques can be used to get results about this. But maybe it is better to find a way to do double-counting using matrices. And I will talk about this as well.