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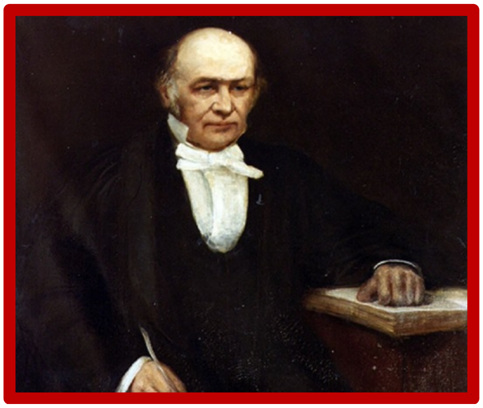
Title: The Quaternions: An Algebraic and Analytic Exploration



Sir William Rowan Hamilton (1805-1865)

	1	i	j	k	-1	-i	-j	-k
1	1	i	j	k	-1	-i	-j	-k
i	i	-1	k	-j	-i	1	-k	j
j	j	-k	-1	i	-j	k	1	-i
k	k	j	-i	-1	-k	-j	i	1
-1	-1	-i	-j	-k	1	i	j	k
-i	-i	1	-k	j	i	-1	k	-j
-j	-j	k	1	-i	j	-k	-1	i
-k	-k	-j	i	1	k	j	-i	-1

The Cayley table for the quaternion group

Abstract: Sir William Rowan Hamilton introduced the quaternions on October 16, 1843 as a "four dimensional number system." The quaternions have additive and multiplicative identities and inverses. Addition and multiplication are associative and addition is commutative. However, multiplication is not commutative. In modern terminology, the quaternions are a noncommutative division ring. Without commutivity, we lose the Factor Theorem and we get the curious behavior that an nn degree polynomial may not have nn roots; for example, the polynomial $p(qq) = qq2 + 1$ has an infinite number of roots. We explore the roots of polynomials of a quaternionic variable, state a Fundamental Theorem of Algebra, and resolve the relationship between the number of roots and the degree of a polynomial. Next, we explore an analytic theory of functions of a quaternionic variable. We describe several results from classical complex analysis which have been extended to the quaternionic setting (namely, the Maximum Modulus Theorem, the Gauss-Lucas Theorem, and Bernstein's Inequality).