Some theorems and conjectures about extremal finite set structures

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I shall discuss a number of recent analogues of two theorems of mine from 1976. One of these, generalizing the Erdos-Ko-Rado theorem, is that if $A_1, \ldots, A_t$ are $t$ cross-intersecting families of $k$-subsets of $\{1, 2, \ldots, n\}$, where $k \leq \frac{n}{2}$, then

$$|A_1| + \ldots + |A_t| \leq \max \left\{ \binom{n}{k}, t \binom{n-1}{k-1} \right\}.$$

Here “cross-intersecting” means that if $A_i \in A_i$ and $A_j \in A_j$ and $i \neq j$, then $A_i \cap A_j \neq \phi$ (not empty).

The other is that if $A$ and $B$ are two mutually strictly incomparable complement-free families of subsets of $\{1, 2, \ldots, n\}$, then

$$|A| + |B| \leq 2^{n-1}.$$

Here “$A$ is complement-free” means that $A$ does not contain any set and its complement. “$A$ and $B$ are mutually strictly incomparable” means that if $A \in A$ and $B \in B$ then $A \not\subseteq B$ and $B \not\subseteq A$.

The recent analogues are joint work by myself, Jie Zheng and John Goldwasser.