KEYNESIAN THEORY AND POLICY AT A GLANCE
DERIVATION OF THE TAX MULTIPLIER

When the level of taxation increases by some amount, ΔT, the equilibrium level of income will *de*crease by some amount, ΔY. The tax multiplier, first cousin to the investment multiplier, is the (negative) ratio of ΔY to ΔT. It can be derived, as follows, from the equilibrium condition ($Y = C + I + G$) together with the equation defining disposable (i.e., after-tax) income ($Y_d = Y - T$) and the consumption equation as applied to disposable income ($C = a + bY_d$).

1. $Y = C + I + G \quad$ where $C = a + bY_d$ and $Y_d = Y - T$
2. $Y = a + bY_d + I + G$; $\;Y = a + b(Y - T) + I + G$
3. $Y = a + bY - bT + I + G$
4. $Y + \Delta Y = a + b(Y + \Delta Y) - b(T + \Delta T) + I + G$
5. $Y + \Delta Y = a + bY + b\Delta Y - bT - b\Delta T + I + G$
6. $Y \qquad = a + bY \qquad - bT \qquad + I + G$

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7. $\qquad \Delta Y \quad = \qquad b\Delta Y \qquad - b\Delta T$
8. $\Delta Y - b\Delta Y = - b\Delta T$
9. $(1 - b)\Delta Y = - b\Delta T$
10. $\Delta Y = \dfrac{- b\Delta T}{(1 - b)}$ $\qquad$ or 11. $\dfrac{\Delta Y}{\Delta T} = \dfrac{- b}{(1 - b)}$

The 10-step derivation above consists of the following sequence of manipulations:

1. Write the equilibrium condition letting it describe the initial equilibrium.
2. Replace C in this equation with its algebraic equivalent, $a + bY_d$, and then replace Y_d with its definition, $Y - T$.
3. Remove the parentheses in step 2, algebraically.
4. Rewrite equation 3 substituting $Y + \Delta Y$ for Y and $T + \Delta T$ for T. This equation describes the new equilibrium once the economy has adjusted to the increase in the level of taxation.
5. Remove the parentheses in step 4, algebraically.
6. Rewrite equation 3 aligning the corresponding terms.
7. Subtract equation 6 from equation 5.
8. Transpose $b\Delta Y$ to the left side of the equation.
9. Factor out the ΔY.
10. Divide both sides of the equation by $(1 - b)$. This equation tells us that if we know that the level of taxation has increased by ΔT, we can multiply by $-b/(1 - b)$ to determine the corresponding magnitude of ΔY.
11. Alternatively, divide both sides of this equation by ΔT to get the defining statement of the tax multiplier. Note that the tax multiplier is the negative of the ratio of the marginal propensity to consume to the marginal propensity to save.