KEYNESIAN THEORY AND POLICY AT A GLANCE
DERIVATION OF THE INVESTMENT MULTIPLIER

The notion of an investment multiplier is most relevant when (1) the economy is functioning somewhere below its full-employment level and (2) market forces, which normally impinge on prices, wages and the interest rate, are (for some reason) not working. In these circumstances, a (Keynesian) macroeconomic equilibrium (one involving a substantial amount of economywide unemployment) is achieved through changes in the levels of spending and income.

When the level of investment increases by some amount, ΔI, the equilibrium level of income will increase by some multiple amount, ΔY. The ratio of ΔY to ΔI is called the investment multiplier. It can be derived, as follows, from the equilibrium condition ($Y = C + I + G$) together with the consumption equation ($C = a + bY$).

1. $Y = C + I + G$ where $C = a + bY$

2. $Y = a + bY + I + G$

3. $Y + \Delta Y = a + b(Y + \Delta Y) + I + \Delta I + G$

4. $Y + \Delta Y = a + bY + b\,\Delta Y + I + \Delta I + G$

5. $Y \quad\quad = a + bY \quad\quad + I \quad + G$

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6. $\quad \Delta Y = \quad\quad b\,\Delta Y \quad + \Delta I$

7. $\Delta Y - b\,\Delta Y = \Delta I$

8. $(1 - b)\Delta Y = \Delta I$

9. $\Delta Y = \dfrac{\Delta I}{(1 - b)}$ or 10. $\dfrac{\Delta Y}{\Delta I} = \dfrac{1}{(1 - b)}$

The 10-step derivation above consists of the following sequence of manipulations:

1. Write the equilibrium condition letting it describe the initial equilibrium.
2. Replace C in this equation with its algebraic equivalent, $a + bY$.
3. Rewrite equation 3 substituting $Y + \Delta Y$ for Y and $I + \Delta I$ for I. This equation describes the new equilibrium, once the economy has adjusted to the increase in the level of investment.
4. Remove the parentheses in step 4, algebraically.
5. Rewrite equation 3 aligning the corresponding terms.
6. Subtract equation 5 from equation 4.
7. Transpose $b\,\Delta Y$ to the left side of the equation.
8. Factor out the ΔY.
9. Divide both sides of the equation by $(1 - b)$. This equation tells us that if we know that the level of investment has been increased by ΔI, we can multiply by $1/(1 - b)$ to determine the corresponding increase (ΔY) in the level of income.
10. Alternatively, divide both sides of this equation by ΔI to get the defining statement of the investment multiplier. Note that the investment multiplier is simply the reciprocal of the marginal propensity to save.