## Math 2650 - Linear Differential Equations

## A. J. Meir

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## **Example: Pendulum**

An ordinary differential equation is an equation which involves an unknown function and its derivatives with respect to one independent variable. (A partial differential equation involves an unknown function and its partial derivatives with respect to several independent variables.)

The differential equation (initial value problem) describing a pendulum

$$m\frac{d^2}{dt^2}\phi + \frac{c}{l}\frac{d}{dt}\phi + \frac{m}{l}\frac{g}{s}\sin(\phi) = F(t) \qquad \phi(0) = \phi_0 \qquad \frac{d}{dt}\phi(0) = \phi_1$$

we usually approximate this nonlinear equation, by the linear equation  

$$m\frac{d}{dt}\phi + \frac{c}{l}\frac{d}{dt}\phi + \frac{m}{l}\frac{g}{dt}\phi = F(t) \qquad \phi(0) = \phi_0 \qquad \frac{d}{dt}\phi(0) = \phi_1$$

which is a good approximation when the angle  $\phi$  is small. Here *m* is the mass of the bob, *c* the amount of damping, l length of pendulum, and g acceleration of gravity. The initial condition  $\phi_0$  is the initial angle and  $\phi_1$  the initial velocity. Below we assume F(t) = 0.

## ☐ Maple commends

```
> restart:with(plottools):with(plots):
```

Warning, the name changecoords has been redefined

Warning, the previous binding of the name arrow has been removed and it now has an assigned value

- > pendulum:=proc(m,c,l,g,T,phi0,phi1)
- > local eq, ic, soln, p, L, C, N, seq1, seq2, h, support, bob:
- > global n, x, y, graph:
- > eq:=m\*diff(phi(t),t,t)+(c/l)\*diff(phi(t),t)+(m\*g/l)\*phi(t)=0:

```
> ic:=phi(0)=phi0,D(phi)(0)=phi1:
```

```
> soln:=rhs(dsolve({eq,ic},phi(t))):
```

```
> graph:=plot({soln,diff(soln,t)},t=0..T,thickness=2):
> p:=unapply(soln,t):
```

```
> L:=(x,y)->line([0,0],[x,y],thickness=3,color=red):
> C:=(x,y)->disk([x,y],0.3,color=blue):
> N:=200:
> x:=l*sin(p((T/N)*n)):y:=-l*cos(p((T/N)*n)):
> seq1:=seq(L(x,y),n=0..N):
> seq2:=seq(C(x,y),n=0..N):
```

- > h:=line([-4,0],[4,0],thickness=5,color=green):
- > support:=display(seq1,insequence=true):
- > bob:=display(seq2,insequence=true):
- > display(h,support,bob,scaling=constrained):
- > end:

```
> Mass:=1/20:Damping:=0.03:Length:=4:G:=10:Time:=20:
InitAngle:=1:InitVelocity:=-1:
```

> pendulum(Mass,Damping,Length,G,Time,InitAngle,InitVelocity);graph;





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