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Math-5630/6630
Introduction to Numerical Analysis I
Summer 2007

Homework 10

Problems

1. Do problem 2 on p. 532 of your textbook, repeat also for the backward Euler method.

2. Use Euler's method to solve:

(a) $$y' = y\,, \qquad y(0) = 1\,.$$

(b) $$y' = 1 + (y - t)^2\,, \qquad y(0) = 1\,.$$

Find $y(0.2)$ using $h = 0.1$. Compare to the exact solutions which are $y = e^t$ and $y = t + \frac{1}{1-t}$, respectively.

Program

1. Program Euler's method.

2. Program Heun's method.

3. Program fourth order Runge-Kutta method.

Apply all methods, using $h = 0.1$ to the equations:

$y'(t) = y(t)\,, \quad 0 \le t \le 3\,, \qquad y(0) = 1\,, \qquad$ which has the exact solution $\quad y = e^t\,.$

$y'(t) = y(t) - 1\,, \quad 0 \le t \le 3\,, \qquad y(0) = 0\,, \qquad$ which has the exact solution $\quad y = 1 - e^t\,.$

Format the output in a table with following headers:

t_i	Euler	Error	Heun	Error	RK-4	Error

For the errors use the absolute value of the difference between the approximate solution and the exact solution.
Extra. Try to repeat the above using $h = 0.1$ for the fourth order Runge-Kutta method, $h = 0.05$ for Heun's method, and $h = 0.025$ for Euler's method.